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# NEW STOCHASTIC MODEL APPLIED IN ASSESSMENT OF THE FINANCIAL DISTRESS

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**Abstract:** In this paper, based on the research study performed by Krysiak and Seaman in 2012, we are trying to develop the stochastic model which would be applied for identification the level of financial distress within the enterprises caused by the banking sector. The shifting of risk phenomenon between financial and non-financial enterprises creates a kind of distress on the side of non-financial institutions leading to a chain of bankruptcies. Risk-shifting does not have anything to do with risk transfer for hedging or risk mitigating purposes. The paper is focused on new stochastic models, which would be applied to identify the circumstances at which arises the danger for huge number scale of bankruptcies within the enterprise sector, what can lead as well to the crisis on a big scale.

**Keywords:** Risk-Shifting; Risk Transfer; Volatility Index; Implied Volatility; Historical Volatility; Stochastic Process; Triangular Transformation.

**2010 AMS Subject Classification:** 62P20.

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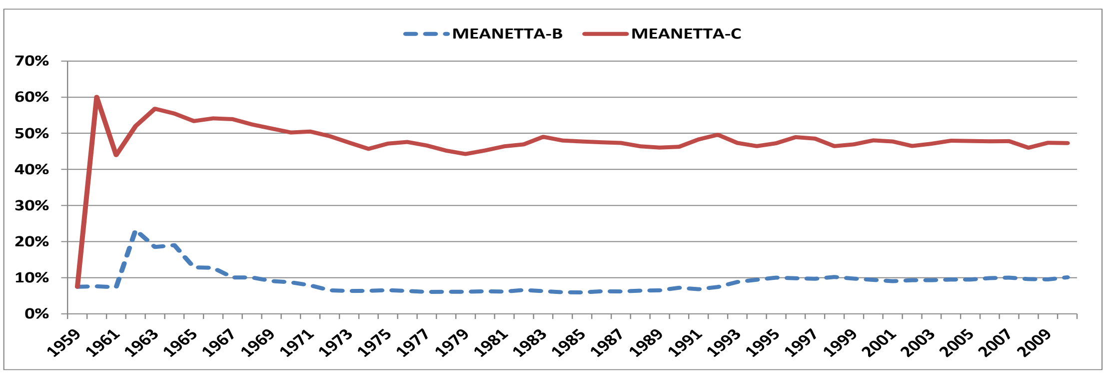
# 1. Introduction

## 1.1 Impact of banking sector on enterprise's economic condition in the USA.

The economic conditions of the companies in the USA were observed to be impacted by banking sector. We want to utilize conclusions obtained in the research study performed by Krysiak and Seaman in [9]. In the above mentioned research authors examined financial reports data (profit and loss statement and balance sheet), for banks and enterprises from the Wharton database. The analyzed data were derived from the period 1959 - 2010, and were composed of 800 banks and around 4500 enterprises from real economy sector. The primary hypothesis stated by Krysiak and Seaman [9] maintains that the banks' funds, allocated to projects in the real economy, cannot yield returns which consistently exceed those realized by the borrowing enterprises, and that the risk shared between the banking and enterprise sectors cannot be, on average, extremely disproportional.

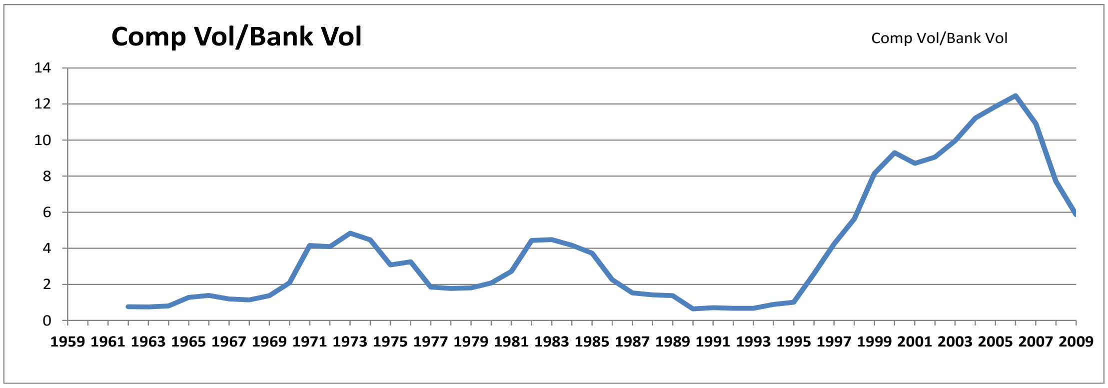
**Figure 1:** The Ratios: Value of Equity to Total Assets for Banks and Enterprises




Source: Krysiak, Seaman 2012

Figure 1 presents value of capital in relation to total assets for banks and enterprises. On average value of equity at enterprises approaches 50% of total assets but at the banks it is only on the level between 6%-10%. The level of equity, in both sectors, reflects the difference in risk assumed by each. Assuming that the relationship between risk amongst enterprises and the risk amongst banks should be proportional, we can be surprised by the results shown in figure 2, where is observed that after 1998 the above mentioned relation is inexplicably large approaching, on occasion, a factor of 12. The volatility ratio of enterprises risk over the banks risk systematically increases after LTCM crises and crises in Russia up to the 2008 when we observed explosion global financial crisis triggered in the USA.

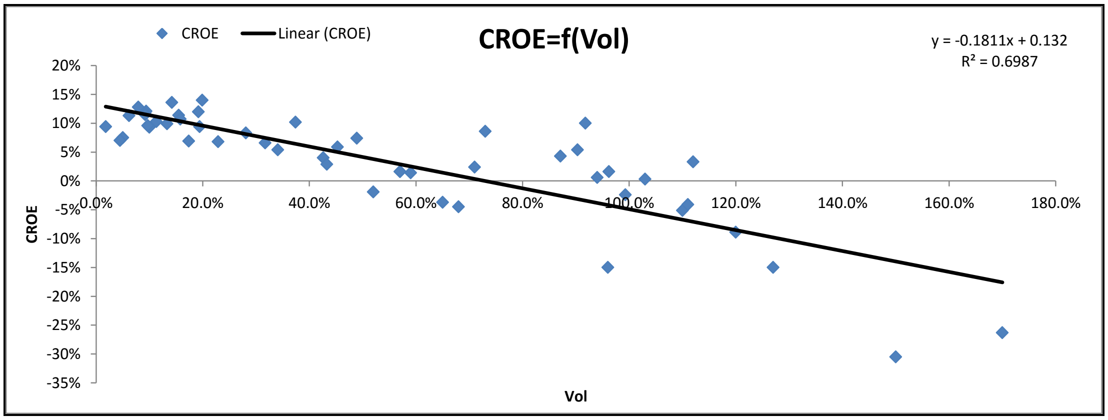
**Figure 2:** Relationship between the Enterprises and Banks Risk




Source: Krysiak, Seaman 2012

Figure 2 illustrates the relationship between the risk charged to banks and enterprises. From 1970 to 1996 the risk charged to enterprises was between 1-4 times higher than the risk charged to banks. From 1970 up to 1996, according to Krysiak and Seaman [9], it was observed interchangeable cyclicality in volatility across both sectors. After 1996 there is a substantially greater risk on the enterprise side than on the bank side. The significant difference in risk in favor of banks begins in 1997. In 1999 there was almost 8 times higher risk on the enterprise side than the bank sector. This difference in risk could be assumed as a very early warning signal of financial crisis. This pattern indicates that early warning of financial crisis was available at least a few years before crisis began.

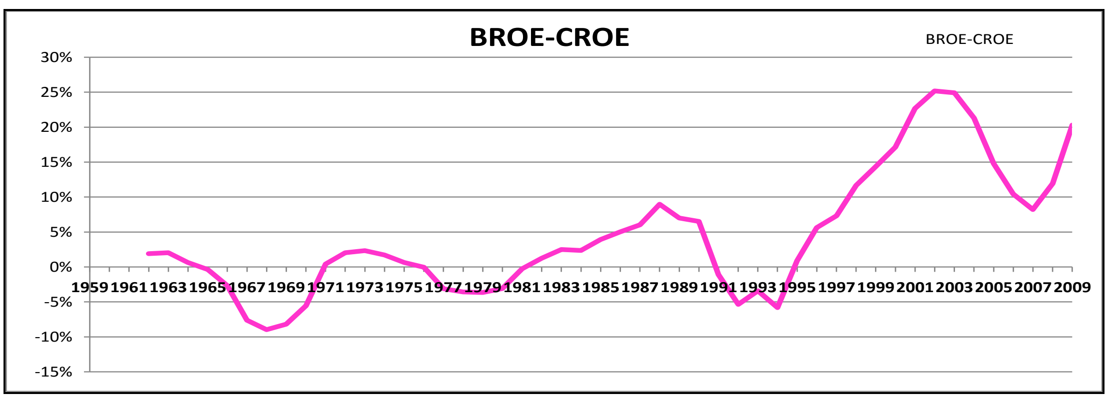
**Figure 3:** The Regression between the Enterprise's Return on Equity and its Risk




Source: Krysiak, Seaman 2012

Figure 3 illustrates a strong dependence of the return on equity from risk expressed by volatility of return on equity. This supports the hypothesis that risk transferred by banking sector into the enterprise sector reduces yields of companies. The declining profits and yields on the enterprise side should raise the concern of banks as bankruptcy of a bank's customers may trigger the bank's default.

**Figure 4:** The Difference in the Return on Equity between the Banks and Enterprises

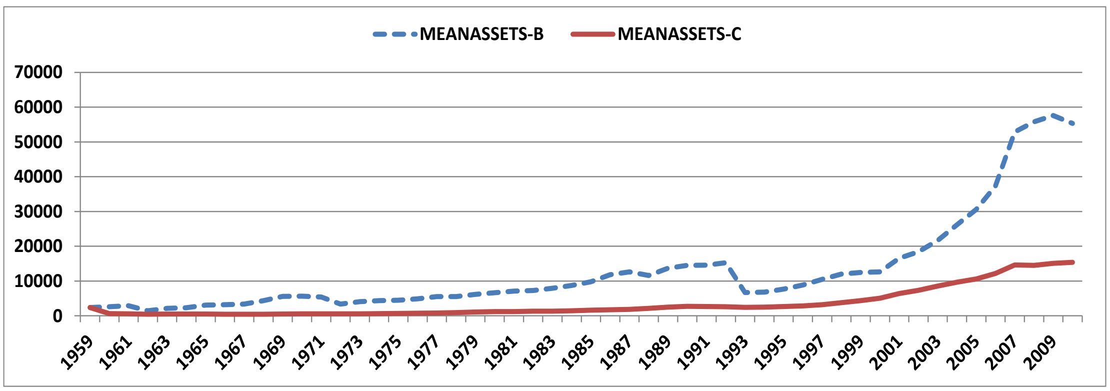



Source: Krysiak, Seaman 2012

Figure 4: provides the difference in returns gained by banks over the enterprises. This difference was periodically favorable for enterprises or banks from 1970 up to 1996. After 1996

were observed extreme differences favoring banks (Krysiak, Seaman, [9]). These differences exceed the level of 10 % on average in favor of banks, and at the extreme in period 1999-2005 it is over 20%. This very significant difference in return on equity beginning from 1999 could be an early warning signal of financial crisis. Assuming that the difference of about 5% is justified, a difference in ROE approaching 10% in favor of banks might offer early warning of crisis.

**Figure 5:** Average Total Assets Value for the Banks and Enterprises




Source: Krysiak, Seaman 2012

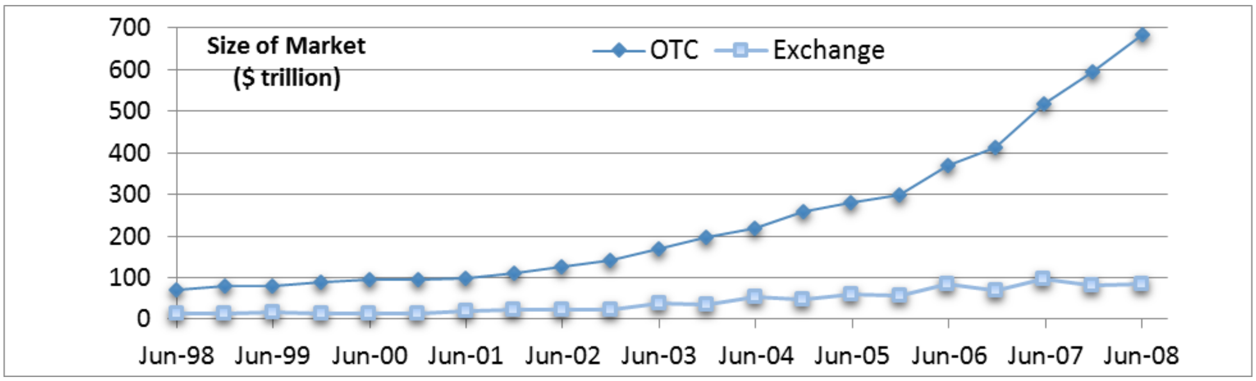
Figure 5 presents the average total assets value for the banks and enterprises. From 1996 we observe increasing spread between the banks and enterprise assets value. If the total assets value can be treated as a proxy of the company value then it would imply that with the risk transfer from the financial sector into real economy, there is an associated transfer of value and gains. The risk transfer from banks to the enterprises results in asset's value increase on banks side with devaluation of companies' value. The devaluation of assets value could be interpreted as an impact of discussed mechanisms in previous sections.

## 1.2 Modeling the risk-shifting based on data observed on derivatives market

Can the volatility swap's market and subsequently the derivative market as a whole explain the risk-shifting by pushing down the equity value and the return on equity?

The global derivatives market value is around $900 trillion, whereas the global GDP is approximately about $60 trillion. The USA annual GDP equals to $15 trillion and the USA debt is equal with GDP. Global derivatives value is 15 times higher than to the global GDP. This ratio seems to be increasing and we think that it has negative impact on the government debts around the world.

**Figure 6:** Growth of Global Derivatives Markets




Source: Bank for International Settlements

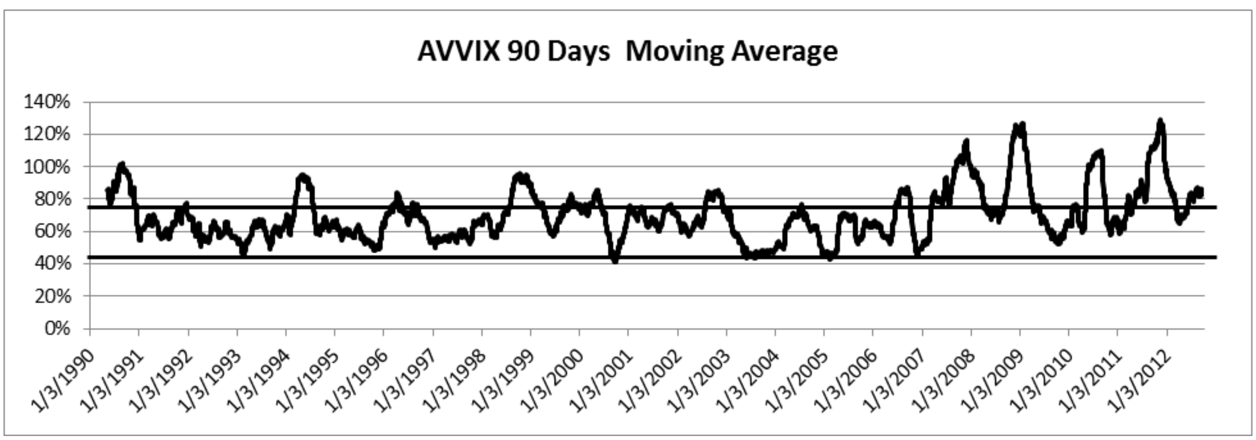
The indicators formulated by Krysiak and Seaman [9] were observed on the abnormal level since the 1998, what is correlated with the period of haste growth in the value of global trade on derivative market. This process is shown in the Figure 2: and Figure 6.

The Chicago Board Options Exchange (CBOE) Volatility Index, VIX, calculated based on prices of out-of-the-money put and call options on the S&P 500 index (SPX), SPX and the historical realized volatility shows interesting correlations. Hsu, Murray [7], presents interesting correlations for the SPX, VIX, and the historical realized volatility of SPX for period 2 Jan 1990 to 29 Jun 2006. Biscamp and Weithers [1], presents the long time series of VIX and historical realized volatility of SPX with evidence of long term difference between VIX and HIX (Historical Realized Volatility on SPX) for period 1 Feb 2006 to 1 Aug 2007.

We claim that constant difference between the historical and implied volatility in favor of the latter creates the systematic process of risk-shifting from financial institutions to the enterprise's equity. This leads to decline in the equity value. The difference between volatilities results in settlement of the volatility SWAP contracts on the market what for the "equity" means taking long position (looser position) and for the financial institutions traders means taking short position (winner). That kind of long term systematic relationship cumulates losses in capital on enterprises side and cumulates gains on the financial institution side.

This mechanism is triggered by the Volatility SWAP market but it is then transferred on to the Equity Options, Credit Derivatives used widely by banks, insurance companies and hedge funds which corresponds to the enterprise's funding structure (relationship between the equity and debt), convertible bonds (result of the change in the enterprise risk level), IRS (Interest Rates Swaps), CDOS (Collateral Debt Obligations Swaps), and finally Foreign Exchange Derivatives Instruments since the exchange rate market is closely corresponding to the trade on the financial and capital markets.

**Figure 7:** Annual volatility of VIX index



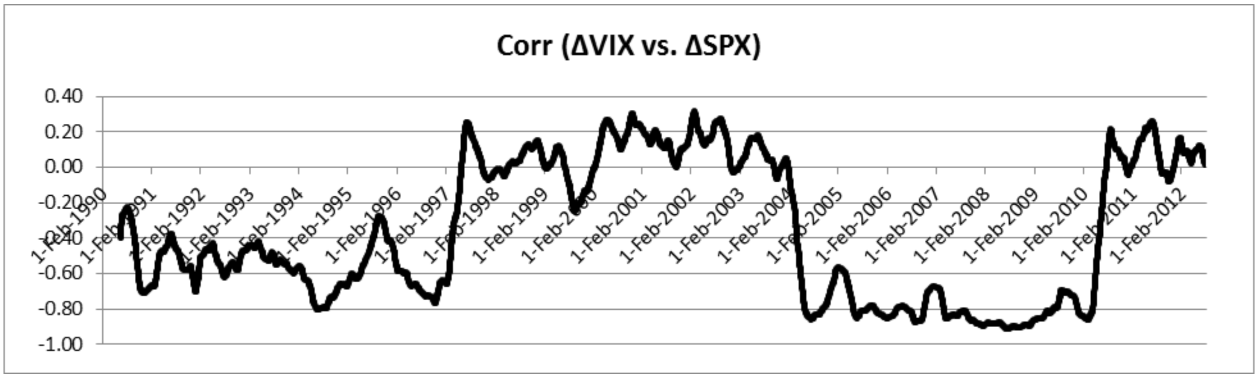

Source: The computation made by Authors based on data from Yahoo Finance regarding VIX index from CBOE

Figure 7 shows annual volatility of VIX index. We can observe that on average since 2008 the volatility of VIX is much over the level observed before. The crisis in Europe and the fragile situation in the USA economy correlated with high level of sovereign debts around the globe, and rising high value of derivatives can be strongly correlated with the indicators we considered in that paper.

The very common view of the traders on the derivative market is confidence in constant high negative correlation between the change in VIX index and change in SPX index. We illustrate on the Figure 8 that this is not true and yet this correlation evolves cyclically from positive to negative values. Variation in SPX and VIX correlations subsequently alters the cash flows obtained in the transactions between the financial sector and real economy sector. In this case application of different, than in reality, correlations into the valuation of derivatives will impact very much the valuation of the real assets expressed in traded shares. The relationship between SPX, VIX and HIX indexes can be treated as a kind of stochastic process. Based on this assumption we can model the amount of risk-shifting phenomenon by identifying the value when the stochastic process jumps over certain level.

The probability of that jump over can be assumed as a measure of financial distress. In order to handle these indicators, in the following sections we construct and develop some, rather new, stochastic models. The models are stochastic processes defined by sequences of the triangular transformations defined by Filus, Filus and Arnold [6] (also, see Filus [2] ). Each such an $R^n \rightarrow R^n$ transformation (n = 2, 3, … ) maps an n dimensional random vector to another. As $n \rightarrow \infty$, one obtains a $R^\infty \rightarrow R^\infty$ transformation of, in general simple, discrete time stochastic process into, so constructed, new stochastic process. A new example of application of this method for the Pareto distribution case is given in section 4. In section 5 the construction procedure is generalized for the continuous time stochastic processes and then applied in modeling the return on equity for banks and enterprises to make a comparison between them. For the prediction purposes we use a proper approximation of the stochastic integral from the basic stochastic process over time interval from presence to some time epoch ahead. We finally arrive with the conclusions on relationships between risk levels of the banks and the corresponding enterprises.

**Figure 8:** Correlation between the change in VIX index and change in SPX index




Source: The computation made by author based on data from Yahoo Finance on VIX index from CBOE

## 2. Probabilistic Preliminaries

The following probabilistic model is quite general and can likely be adopted for most of the econometric (random) quantities (see Tsay [10]), say, $X_t$.

In the main case we here consider, $X_t$ will be return on an equity at time t, which may be discrete, taking on the values $t = 0, 1, 2, \dots ,m, \dots$ ; possibly days or months.

The sequence $\{X_t\}$ is then a stochastic process, so that for each $t = 0, 1, \dots$ , $X_t$ denotes a continuous random variable ( r.v.).

Our first aim is to find an analytic pattern for the conditional probability distributions of $X_t$ , given realizations $x_0, x_1, \dots ,x_{t-1}$ (the past) of the rvs. $X_0, X_1, \dots ,X_{t-1}$ for each $t = 1, 2, \dots$ .

At this point, first one should admit that the above task means more than the usual one, where only the conditional expected values $E[X_t \mid x_1, x_2, \dots ,x_{t-1}]$ and, eventually, the conditional variances $Var[X_t \mid x_1, x_2, \dots ,x_{t-1}]$ were aimed to be found. (The, below obtained, stochastic processes, in general, will be nonMarkovian but still simple enough to perform underlying calculations.)

The difference becomes even more essential when $X_t$ or the conditional probability distribution of $X_t$ , given a past, is no more assumed to be the Gaussian.

In the considered below framework, we do not specify a class of the probability distributions of $X_t$ with exception that the underlying probability densities always exist.

All this makes the following investigations essentially new comparing to the existing so far in literature, possibly with exception of Filus and Filus [5] (according to our best knowledge).

## 3. The General Model Construction

We define the probabilistic model using the following procedure. Consider any stochastic process $T_1, \ldots, T_t, \ldots$ . In particular it can be a white noise in the sense that all the underlying r.vs $T_j$ ( $j = 1, 2, \ldots, t$) are independent and identically distributed according to some pdf, say, $f(t_j)$ belonging to an arbitrary class of pdfs..

Then, for each $t = 2, 3, \ldots$ apply to the random vector $(T_1, \ldots, T_t)$ the following pseudoaffine case of the 'triangular transformation' (1) (see, Filus, Filus and Arnold [6] ) that "sends" the random vector $(T_1, \ldots, T_t)$ to the random vector $(X_0, X_1, \ldots, X_t)$:

$$
\begin{aligned}
X_0 &= x_0 \\
X_1 &= A_0(X_0)\, T_1 + B_0(X_0) \\
X_2 &= A_1(X_1)\, T_2 + B_1(X_1) \\
X_3 &= A_2(X_1, X_2)\, T_3 + B_2(X_1, X_2) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
X_{t-1} &= A_{t-2}(X_1, X_2, \ldots, X_{t-2})\, T_{t-1} + B_{t-2}(X_1, X_2, \ldots, X_{t-2}) \\
X_t &= A_{t-1}(X_1, X_2, \ldots, X_{t-1})\, T_t + B_{t-1}(X_1, X_2, \ldots, X_{t-1}), \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}
\tag{1}
$$

where $x_0$ is any, say nonnegative, real number considered as an 'initial value' of the stochastic process $\{X_t\}$ and the first equality in (1) is assumed to hold with the probability one.

All other equalities in (1) can be meant in the sense that the probability distributions of both sides of each equality are the same. Since the transformation (1) is well defined for each t, (1) can be treated as the general scheme which defines the whole stochastic process $\{X_t\}$, given any well defined stochastic process $\{T_t\}$ (the consistency conditions can readily be checked).

Also realize that if each of the (say, independent) random variables $T_t$ is normally distributed then all the resulting random vectors $(X_1, X_2, \dots, X_t)$ $t = 1, 2, \dots$ , have the pseudonormal distribution (see, Filus and Filus [3] and the citation in Kotz, Balakrishnan and Johnson [8].) For the, so obtained, pseudonormal stochastic process $\{X_t\}$, see Filus and Filus [5].

In the same vein, if the pdfs. of $T_t$'s are the exponential, gamma or Weibullian the so obtained joint distributions of $(X_1, X_2, \dots, X_t)$ are pseudoexponential, pseudogamma, pseudoWeibullian respectively (see, Filus and Filus [4] ).

However, as we have recently learned, the name "pseudonormal" was already used in literature for other distributions. Hence we propose to replace it by the new name "FF-normal" or "FF-Gaussian" and for the consistency the other distributions we will name "FF-exponential", "FF-gamma", "FF-Weibullian" respectively.

This new terminology we will use from now on throughout.

Now, realize that the transformation (1) is easily invertible upon the assumption:

$A_{j-1}(X_1, X_2, \dots, X_{j-1}) \neq 0$ with the probability one.

For each $j = 1, 2, \dots, t$ we have:

$$T_j = [X_j - B_{j-1}(X_1, X_2, \dots, X_{j-1})] \;/\; A_{j-1}(X_1, X_2, \dots, X_{j-1}) \qquad (2)$$

If one would adopt the assumption that the common pdf $f(t_j)$ for all the $T_j$ is the standard normal then, according to our interpretation of equalities (1), (2), the right hand side of (2) also has the N(0, 1) pdf. (The latter fact makes a further statistical analysis relatively easy, when sampling the values $t_j$ of $T_j$.)

Also realize, that the jacobian of that inverse (2) reduces to the product:

$J(x_0, x_1, x_2, \dots, x_{t-1}) = [A_0(x_0)\, A_1(x_1) \dots A_{t-1}(x_1, x_2, \dots, x_{t-1})\,]^{-1}$.

From all that, one obtains the conditional pdf

$g_j\,(x_j \mid x_1, x_2, \dots, x_{j-1})$ of the rv. $X_j$ , given that, for some sequence of real values $x_1, x_2, \dots, x_{j-1}$ , random event $(X_1, X_2, \dots, X_{j-1}) = (x_1, x_2, \dots, x_{j-1})$ happens.

Therefore, if for any $t$ the values $x_1, x_2, \dots, x_{t-1}$ are known, then the (conditional) pdf $g_t\,(x_t \mid x_1, x_2, \dots, x_{t-1})$ is known Gaussian

$N(B_{t-1}(x_1, x_2, \ldots, x_{t-1});\ A_{t-1}(x_1, x_2, \ldots, x_{t-1}))$ univariate distribution in $x_t$.

For the conditional expectation of $X_t$, one obtains

$$E[X_t \mid x_1, x_2, \ldots, x_{t-1}] = B_{t-1}(x_1, x_2, \ldots, x_{t-1}), \tag{3}$$

and for the corresponding conditional variance

$$Var[X_t \mid x_1, x_2, \ldots, x_{t-1}] = \{A_{t-1}(x_1, x_2, \ldots, x_{t-1})\}^2. \tag{4}$$

Realize that, in the here considered model, the usual linear (regression) function of the variables $x1, x_2, \ldots, x_{t-1}$ is replaced by any continuous (!) function

$B_{t-1}(x_1, x_2, \ldots, x_{t-1})$ and any continuous function $\{A_{t-1}(x_1, x_2, \ldots, x_{t-1})\}^2$ determines the conditional variance (the heterescedastic case, in general).

The so obtained wide extension of the existing models allows, however, to treat the very well known ones (normal) as special cases of that, above considered, upon the properly stated assumptions.

For example, one obtains back the linear regression model if one assumes in (3) that

$B_{t-1}(x_1, x_2, \ldots, x_{t-1}) = c_1x_1 + c_2x_2 + \ldots + c_{t-1}x_{t-1}$.

One also obtains a typical (nonheteroscedastic) model if one, properly, choses

$A_{t-1}(x_1, x_2, \ldots, x_{t-1}) = A_{t-1}$ to be a constant.

## 4. Example of the Model Application: Multivariate ‘FF-Pareto’

In this section we (temporarily) abstract from particular economical meaning of the considered random variables.

Consider, as an example, the following “pseudolinear” part of the pseudoaffine transformation (1) (with the symbols $T_j$ replaced by $X_j$ and $X_j$ replaced by $Y_j$) which one obtains by setting in (1) all the “pseudotranslation” coefficients

$B_{j-1}(X_1, X_2, \ldots, X_{j-1})$ to zero. We then have the transformation:

$Y_0 = y_0$

$Y_1 = A_0(Y_0)\, X_1$

$Y_2 = A_1(Y_1)\, X_2$

$Y_3 = A_2(Y_1, Y_2)\, X_3$

…………………………………………

$Y_{t-1} = A_{t-2}(Y_1, Y_2, \dots, Y_{t-2})\, X_{t-1}$

$Y_t \quad = \quad A_{t-1}(Y_1, Y_2, \dots, Y_{t-1})\, X_t$

……………………………………………………………………… (5)

$t \rightarrow \infty$.

We investigate how the latter transformation acts on set of independent Pareto distributed rvs.

$X_j$

$( j = 1, 2, \dots, t; \quad t = 1, 2, \dots )$ in (5).

Recall, the (here considered) Pareto density is given as:

$$f_j(x_j) = 1 / \beta\, (1 + x_j / \beta\gamma\, )^{1+\gamma}, \tag{6}$$

where $\beta$ and $\gamma$ are positive real parameters.

According to (5), for every $j = 1, \dots, t$, we may express $x_j$ as

$x_j = y_j / A_{j-1}(y_1, y_2, \dots, y_{j-1})$, upon the assumption $A_{j-1}(y_1, y_2, \dots, y_{j-1}) \neq 0$.

Also realize, that the jacobian of inverse to (5) equals to the product:

$J(y_0, y_1, y_2, \dots, y_{t-1}) = [A_0(y_0)\, A_1(y_1) \dots A_{t-1}(y_1, y_2, \dots, y_{t-1})\,]^{-1}$.

As next step, one obtains ( for ach $j = 1, 2, \dots, t$ ) the conditional pdfs $g_j( y_j \mid y_0, y_1, \dots, y_{j-1})$ of each rv. $Y_j$, given the realizations (i.e., the observed prices of the assets) $y_0, y_1, \dots, y_{j-1}$ of the rvs. $Y_0, Y_1, \dots, Y_{j-1}$, as follows:

$g_j( y_j \mid y_0, y_1, \dots, y_{j-1}) = f(x_j)\, | \partial x_j / \partial y_j | =$

$f( y_j / A_{j-1}(y_1, y_2, \dots, y_{j-1})\, )\, |A_{j-1}(y_1, y_2, \dots, y_{j-1})\, |^{-1}$

$$= 1 / \{ \beta\, |A_{j-1}(y_1, y_2, \dots, y_{j-1})| \quad [\, 1 + y_j / \beta\, |A_{j-1}(y_1, y_2, \dots, y_{j-1})|\, \gamma\, ]^{1+\gamma} \} \tag{7}$$

So, the effect of each (j+1)-th line of the transformation (5) on the rv. $X_j$ is to change its

Pareto density (6) for the (conditional) Pareto density (7) of $Y_j$ .

The two Pareto densities (6), (7) only differ by the scale parameters, so:

$\beta$ in (6) was transformed into the product ' $\beta \mid A_{j-1}(y_1, y_2, \ldots ,y_{j-1}) \mid$' in (7).

The latter relation "visualizes" the effect of "past values" $y_1, y_2, \ldots ,y_{j-1}$ on the (still Pareto) probability distribution (7) of the future value $Y_j$.

Here, we assume that the present time is at $t = j-1$.

Two important facts follow. Firstly, even after estimating any of the involved parameters, the observed values $y_1, y_2, \ldots ,y_{j-1}$ determine only the (conditional) density (7) of the random variable $Y_j$ and not its exact value $y_j$ . Secondly, the simplicity of the analytical expressions such as (7) allow to drop the usual Markovianity assumption ! (Filus and Filus [5] ).

As for the choice of the "parameter function" $A_{j-1}(y_1, y_2, \ldots ,y_{j-1})$ it is up to a given practical situation and corresponding data.

As a "candidate" we should chose (by an 'educated guess') a parameter class of continuous ( not necessarily linear or quadratic only **!**) functions in $\mathbf{y_1, y_2, \ldots ,y_{j-1}}$ such as, for example, the following 'log-linear'

$A_{j-1}(y_1, y_2, \ldots ,y_{j-1}) = C \exp[ c_1y_1 + \ldots +c_{j-1}y_{j-1} ]$

or any other class of the functions whose members are identified by several real parameters. The parameters are to be estimated by, for example, maximum likelihood estimator or other.

Each such a candidate class of the functions determines a separate '(sub)model' . After estimating all the model's parameters, we need statistically discriminate among several such the (sub)models to choose the best one, in the sense of fit to a given data.

As for the Pareto density's shape parameter $\boldsymbol{\gamma}$ it remains invariant under the pseudolinear transformation (5).

## 5. Continuous Time and Markovian Version of the Model

We also construct stochastic models similar to these defined by (1). This time, however, the obtained stochastic processes will be Markovian but with the continuous time. The processes defined below we apply to the economic problems formulated in section 1. We define them by the following class of "continuous pseudoaffine transformations":

$X_0 = x_0$

………..

$X_{t-\tau} = A_0 (X_0, 0)\, T_{t-\tau} + B_0 (X_0, 0)$

……………………………………………………………..

$X_t = A_{t-\tau} (X_{t-\tau}, t-\tau)\, T_t + B_{t-\tau} (X_{t-\tau}, t-\tau)$ (8)

...................................................................

$0 \le t < \infty$, under the '<u>continuity assumption'</u>: If $\tau \to 0$ then

$A_{t-\tau} (X_{t-\tau}, t-\tau) \to A_t (X_t, t)$ and $B_{t-\tau} (X_{t-\tau}, t-\tau) \to B_t (X_t, t)$ for every $t < \infty$ (see, Filus and Filus [5] ).

$X_t$ , in general, can be an 'econometric quantity' of an interest at time epoch t. For example, such quantity may be a 'stock level' at time t as well as an 'inflation rate' of currency, 'employment level' at t etc.

## 5.1 Model Specification

In our specific framework, we consider the $X_t$ to be return on equity of banks which lend some funds to an enterprise(s). For that return we may specify model (8), for example, by assuming:

$A_{t-\tau} (X_{t-\tau}, t-\tau) = \sqrt{\tau}\; X_{t-\tau}$, and

$B_{t-\tau} (X_{t-\tau}, t-\tau) = X_{t-\tau}$ . (9)

In simpler ('Wiener-like') version one may assume in (9) that

$A_{t-\tau} (X_{t-\tau}, t-\tau) = \sqrt{\tau}$. (9*)

As already mentioned, a proper choice of the model **(** i.e., choice of the functions $A_{t-\tau}(\ ,\ )$ and $B_{t-\tau}(\ ,\ )$ **)** is dictated by statistical analysis of an underlying data preceded by an educated guess**.**

We will use the later version of the stochastic model, as given by (8) and (9) or (8) and (9*), to define some 'prognostics measure'_ (of the '*risk*' defined below)
by means of the following (mean value of ) stochastic integral:

$$Y_s = (1/s) \int_t^{t+s} X_u \, du, \qquad (10)$$

where $X_u$ is defined by (8) (if to replace the time parameter t by symbol '**u'**). Time **'t'** in (10) is understood as present time while we aim to predict the defined situation in a time period '**s'** ahead, given a chosen value **s** (one month, for example)**.**

[ The choice of above '*prognostic measure*' as the quantity proportional to the (stochastic) integral from $X_u$ we motivate by the fact that, in many cases, a meaningful notion of "utility" of a given econometric quantity expresses itself as 'the product of the quantity and time of its duration'. Thus, a value of the 'returns on equity ', we consider, also depends on a time amount they are available. ]

Based on (10) and given a fixed value of s, we define the risk as the probability
$p_s = P(Y_s \le c_s)$, where $c_s$ is a predetermined "critical level".

Since the integral (10) is random variable $Y_s$ we pursue **(**instead, like in deterministic case, of finding (10) as a single number**)** to obtain a set of numbers, say, $\mathbf{y_1, y_2, \ldots, y_n}$ which are independent realizations of $Y_s$ (that set of the numbers will play role of a "simple random <u>sample"</u> from $Y_s$ ). The purpose for it is to estimate the probability distribution of the integral $Y_s$.

That, in turn, will allow to estimate the defined above risk
$p_s = P(Y_s \le c_s)$, where the "critical level" $c_s$ is interpreted as "situation of crisis".

In turn, if the risk $p_s$ exceeds some (fixed by a reasonable convention) value $v_s$

(possibly, say, $v_s = 0.10$) then the “critical situation” (the prediction of the ‘crisis danger’ (not yet the ‘crisis’ itself): s time units ahead with the critical probability $v_s$ ) should be declared and, in parallel, a proper action should be taken.

## 5.2 Calculations

Now we show the way of the underlying calculations. First, we perform the calculation of n values $y_1, y_2, \dots, y_n$ (each value independently from others ) of the stochastic integral $Y_s$ given by (10). For this aim we estimate n times the integral by its Riemannian sums as follows. As a partition of the time interval [t, t+s] we chose the division of it on N equal subintervals, say,

[t, t + s/N ), [t + s/N, t + 2s/N), … ,[t + ks/N, t + (k+1)s/N ), … , [t + (N-1)s/N, t+s ].

Next, assuming that N was chosen sufficiently large we estimate

$$\int_t^{t+s} X_u \, du \cong \Sigma_{k=1}^{N} X_{(t+ks/N)} (s/N) = s\, y_j, \qquad (11)$$

j = 1, 2, … ,n.

What is left to be done (given a value **j**) is to find all realizations (values) $x_{(t+ks/N)}$ of the random variables $X_{(t+ks/N)}$ .

The realizations one obtains independently repeating the same simulation procedure for each j = 1, 2, … ,n.

Actually, we only need to simulate realizations $t_t$ of the random variables $T_t$ using the standard sampling methods. As we have assumed, all the $T_t$ are independent and have the same known probability distribution (for example, it can be the normal N(0, 1) pdf. ).

To better see how the values $x_{(t+ks/N)}$ are obtained from the corresponding sampled values $t_{(t+ks/N)}$ , rewrite the transformations (8) in the following form (for realizations):

$$x_{(t+ks/N)} =$$
$$= A_{(t+(k-1)\, s/N)} (\mathbf{x}_{(t+(k-1)\, s/N)}, (t+(k-1)\, s/N))\, \mathbf{t}_{(t+ks/N)} +$$
$$B_{(t+(k-1)\, s/N)} (x_{(t+(k-1)\, s/N)}, (t+(k-1)\, s/N) ). \qquad (12)$$

Now realize that besides the value $t_{(t+ks/N)}$ obtained by sampling from a known probability distribution, we also know each time the, earlier calculated, value $x_{(t+(k-1)\ s/N)}$, from every previous step as it is to be included in the corresponding computer algorithm (the recurrence procedure). That algorithm can easily be constructed from the current simple analytical pattern.

The functions $A_{(t+(k-1)\ s/N)}(\ )$ and $B_{(t+(k-1)\ s/N)}(\ )$ of the transformation also are known. It is then enough to determine the value $x_{(t+ks/N)}$ at each step that corresponds to each consecutive subinterval of the interval's [t, t+s] division. In turn, the so obtained value $x_{(t+ks/N)}$ will be used in the next step to obtain the next value:

$x_{(t+(k+1)s/N)}$.

Each time j, after all these steps (k = 1, 2, … , N) are performed, one obtains a particular (random) value $y_j$ (j = 1, 2, … ,n) of the Riemannian sum (11) divided by s. Then one repeats all the N steps to obtain next value $y_{j+1}$ of the sample until the last value $y_n$ is obtained. Realize that all the n procedures are identical and are independently performed for j = 1, 2, … ,n.

Thus as a result one obtains the simple random sample $y_1, \dots, y_n$ from the (approximation of ) stochastic value

$Y_s = (1/s) \int_t^{t+s} X_u\, du$.

In the next step one obtains an estimate of the probability

$$p_s = P\,(Y_s \le c_s) \tag{13}$$

as equal to the ratio $p_s = n_p / n$, where $n_p$ is the number of the values $y_j$ such that $y_j \le c_s$.

The "(essential) danger of the crisis" is to be declared if $p_s \ge p_0$, for the obtained value $p_s$ and for some adopted fixed value $p_0$ of the risk, (possibly $p_0 = 0.10$ ).

## 5.3 Final Remarks

First notice that regardless of the possible model specifications (9) or (9*) for the aid of the risk calculation, in the formulas (11), (12), we actually applied the general functions A( ) and B( ). The specification of these two functions depends on the nature of a particular economic problem. It is a separate task that mostly relies on statistical analysis of concrete data.

Secondly consider what follows. As it was discussed in section 1, the risk for the banks (that we have shown the methods of calculation for it above), usually is significantly smaller than risk of the enterprises which borrows funds from those banks. As considered in Figure 2 the ratio, say r, of the two levels of the risk, is given by the ratio of the standard deviations (the volatilities). So, to reflect the risk difference phenomenon in our models we incorporate the ratio r by replacing in (12) the general function

$A_{(t+(k-1)\,s/N)}\,(x_{(t+(k-1)\,s/N)}\,,\,(t+(k-1)\,s/N))$ by the product

‘ $r\,A_{(t+(k-1)\,s/N)}\,(x_{(t+(k-1)\,s/N)}\,,\,(t+(k-1)\,s/N)\,)$ ’.

For the banks we may assume $r = 1$ while for the underlying enterprises $r > 1$.

It follows from (11), (12) that, given a fixed level $c_s$, the probability (13) grows as r grows (when r is incorporated into the formulas as the factor) and therefore sooner reaches the critical value $p_0$.

As shown in Figure 2, in recent few decades, the value r itself grows approaching the values in the range of 8-11. Thus, for the enterprises, the risk danger (i.e., the occurrence of the inequality $p_s \geq p_0$ ) may quite easily be of that magnitude that a preventive action should be taken, even if there was no such a direct danger for the corresponding banks.

The kind of a possible action is another problem which is out of scope of this paper. It seems however that the possible target of an action should be decrement of the value of r through a transfer of the risk from the enterprises back to the banks.

At the end two facts yet should be stressed. First important fact is, that the assumption on independence of the stochastic process $\{T_t\}$ may be relaxed. Instead, as the underlying “randomness mechanism” the Wiener or other stochastic process could be chosen depending on a particular economic situation.

Secondly, it follows from Figure 2 that the risk ratio ‘r’ is dependent on time t. However, in the quantity (11) we used for the risk prediction, the value of “time ahead “ s is assumed to be small enough to consider ‘r’ as a constant.

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