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OPTIMIZING PENSION ASSET & SIMULATED DERIVATIVE IN NIGERIA WITH MINIMUM REQUIRED RETURN

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Abstract: This work focuses on a forecast view point, the expansion gain in view of expanding Nigeria's financial market through exchange traded options product. It further mixed existing an AES portfolio returns with simulated returns of a theoretical call option created from stock price forecast. A mean comparison between AES portfolio returns and AES plus simulated call option return was tested to see if there exist the usefulness of adding more aggressive variable income security in Pension portfolio as a stimulant to higher return. Furthermore, contributor's minimum required return was used in our mathematical formulation as a risk minimization measure. A mathematical model-1 and Model-2 involving 5 and 6 variables respectively, 5 inequality constraints covering regulatory limitations and limitation on scarce resource known as Asset Under Management (AUM), suggested and mathematically shown to be possible through "minimization of risk for a set minimum return" while obeying all regulatory controls as our constraints. Optimized portfolio using TORA and MatLab showed a return of 13.06% from AES portfolio without a mix with simulated call option return. A minimum contributor return demand of 15% was used but it failed to achieve this but returned 13.06%. A mix of the AES portfolio with simulated call option return achieved our contributor 15% minimum return demand. Our test of significance at 95% confidence to infer that there is a difference in rate of return between AES fund manager's and our mathematically optimized returns rejected our null and accepted our alternative hypothesis. We therefore posit that there is a 5% probability that our optimized mixed portfolio may not achieve a higher return than the AES fund manager's portfolio.

Keywords: optimizing pension asset; simulated derivative; minimum required return.

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1. Introduction

2016 marks the Nigerian contributory pension industry's 11th year with Asset under

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Management in excess of N5T. In 2004 the initial Pension Reform Act (PRA 2004) came into force in June 2004. The focus is to achieve well thought out goals which include;

1. Pension to be contributory and fully funded
2. Personalized and portable individual Retirement Savings Account (RSA)
3. The management of pension funds privately and the separation of the functions management of assets
4. Trust in the expertise of investment but on regulation
5. The inclusion of life insurance covers for employees by employer
6. Provision of superior and strict central regulator and supervisor of all pension management and payments.

With the 6 above points and more in view, a well-organized and manned Pension Commission (PenCom) headed (2004 -2012) by Mr Muhammed K Ahmad started off in 2004. After the licensing of the initial 7 Pension Fund Administrators (PFAs) and 3 Pension Fund Custodians (PFCs) in December 2005, the contributory pension industry started off operationally with signing contributor into account in 2006. Operational challenges and obstacles where met with resolvable items resolved. Reactions of stakeholders including and especially the contributing members were regularly logged in for eventual review and rework. Amongst such reviews include;

I. Regulatory approval for resigned contributing members to access 25% of RSA balance after 4 months from date of resignation or termination.

II. Review of compulsory enrolment of employees per private sector employer if 15 and above

III. Review of rate of contribution to the scheme as follows;

a) A minimum of 10% by the employer the as against previous minimum of 7.5%

b) A minimum of 8% by the employer against a maximum of 7,.5% in previous 2004 act

IV. Restriction on the rate of pension fund asset allowing for the use of some fraction of RSA balance as equity contribution for residential mortgage

V. The introduction of pension protection fund etc

In all of these reviews, the need of most contributors is often resident in how much have their contributions added over the period of contribution. This borders on Return on Investment (RoI).

Important of note is that the longest contributor retiring in 2016 has barely contributed for 10 years' operational period of Nigeria's CPS. It is also important to put into account the regulation on the ratios of contributors' exposure to approved asset classes and their securities risk-return measurements before qualifying to be included as a component of pension portfolio. From inception, there has been strict regulation of Nigeria's pension asset portfolio beginning with unit investment across all classes of contributors. Aside strict regulations on securities inclusion into Nigeria's pension portfolio, Pension Fund Administrators (PFAs) seem to have only one motivation to drive high return on investment. This is to acquire more new enrollees-members and increase magnitude of their asset management fees. It is viewed in our opinion, that the PFAs who seem to be the most beneficiaries in terms of profiting from Nigeria's CPS are not researching on the expansion of the industry and how contributors' funds could reach higher returns responsible for cancelling possible inflationary tendencies at retirement and benefits payment. The reason may be found in three parts;

1) Non-participation of contributing members in requesting a target rate of return
2) Non-reduction of asset management fees irrespective of investment managers' performance satisfying contributor' minimum required return, specified periodically by regulation of National Pension Commission (PenCom).
3) Non-expansion of the Nigeria's financial industry by delaying opening of financial derivative market.

Herein, we look into some mathematical models of production output, minimization of risk, and optimization of scarce resources for maximum output.

Optimization of picking route based on backtracking algorithm was apply in [1], where the optimization method under the environment of VC++6.0 was verified. Instead we show using MatLab that portfolio representing AES 2013 with a deficit growth of ₦15.75m representing 3.27% less than the portfolio's full growth potential is found within defined assumptions. This would have been averted if contributors' actually set their targets and investment managers optimize from forecasts of future prices using trend analysis as in [2]. We also recommend that Nigeria's Pension Commission begins working to empower contributors through regulation, to have provisions of requesting and setting reasonable targets of Return on their investments [3].

2. Contributors and Investment Portfolio

In February 2015, a draft suggestion on investment of pension fund assets is suggesting a multi-fund regime which may allow pension assets to be invested across 4 different portfolios satisfying defined-age-distribution of contributing members. Until this is implemented, PFAs will continue to invest on two different portfolios, one for the active Retirement Savings Account (RSA) and the Retirees account (RA). In any of there, we would like a regime that can introduce "a contributor required minimum return demand" to stimulate higher return and challenge investment managers of PFAs to greater efficiency and also as a system to match investment performance to contributors' payment of asset-management-fees.

Table 1: Global limit per Asset Class for AUM

S/No	Asset Class	Global Limit in %
1	Government Security	100
2	Corporate Bond	30
3	Money Market	35
4	Ordinary Shares	25
5	Open/Closed end fund	5

Table 2: Distribution of Asset Under Management (AUM) by Asset Class

Asset Class	Value	Weight in Portfolio	Max limit	Type of return
Equity Stock	76,725,528.00	0.1595	0.25	Variable
FGN Bond	353,444,647.35	0.7345	0.80	Fixed
State Bond	34,393,413.72	0.0715	0.20	Fixed
Corporate Bond	5,056,404.11	0.0105	0.20	Fixed
Money Market	8,028,383.56	0.0167	0.35	Fixed
Cash	3,523,646.33	0.0073	0	None
Total	481,172,022.25			

Source: Portfolio of an AES Member fund as at December 2013

A typical example of a portfolio based on table 1 is in table 2 above. Table 2 displays a portfolio whose total wealth (AUM) is =N=481,172,022.25 with a unit price of =N=1. 509. This means that one unit of this portfolio is selling at this time for N1.51k approximately. Another significant fact is the value of the portfolio relating to its unit price from which we can resolve its accounting units. This is the portfolio we be re-optimizing to achieve a 15% return from a more optimized distribution of assets across the available scarce resources (AUM)

$$\frac{A_T}{U_T} = P_T , \qquad (1)$$

where;

A_T = Asset Under Management as at time T

U_T = No of accounting units at time T

P_T = Price of accounting units at time T

For all $T \ = T, 0 < \ T$

From (1), U_T = (481,172,022.25)/(1.1509) = 318,868,139.33 Units of account

A pension portfolio manager focuses on optimization considering application of any of the following;

1. Minimize risk for a specified return
2. Maximize the expected return for a specified risk
3. Minimize the risk and maximize the expected return using specified risk aversion factor
4. Minimize the risk regardless of the expected return
5. Maximize the expected return regardless of the risk

To achieve any of the above, we look at two basic important focus areas which are;

A. Security and market analysis. By this we access attributes of the entire set of possible investment
B. Creating an optimal portfolio of assets. This involves the determination of the Best risk-return opportunities available from feasible investment portfolio and the choice of best portfolio from the feasible set.

To illustrate (a) and (b) above, let us consider the consolidated portfolio from Table 3 below (Source: Consolidated AES- portfolio December 31, 2013);

Table 3: Historical returns rates of the assets by class

EQUITY RETURN RATE	$E(E_r) =$	0.09
MONEY MARKET RATE	$E(M_r) =$	0.1245
FGN BOND RETURN RATE	$E(B_r) =$	0.1043
STATE BOND RETURN RATE	$E(S_r) =$	0.1300
CORPORATE BOND	$E(C_r) =$	0.118

2. The Mathematical & Mathematical Formulations

We calculated expected returns from portfolio elements in their asset classes. To minimize risk, we need to also compute standard deviation. In this case we applied Harry Markowitz variance or standard deviation as a means of risk measurement [4]. That is;

$$\text{Min} \sum \sum S_{ij} X_i X_j, \qquad (2)$$

Such that the following conditions

1 $\sum_{j=1}^{n} r_j X_j \geq \rho\omega$

2 $\sum_{j=1}^{n} X_j j = \omega$

3 $0 \leq j \leq \mu_j \; j = 1 \ldots n$

Forming with i and j security/asset over a period “T” we have;

$$S_{ij} = \frac{1}{T}\sum_{t=1}^{n}(X_{it} - r_i)\left(X_{jt} - r_j\right) \qquad (3)$$

Equation (3) is the covariance of these securities/assets i and j and

X_{jt} = each security/asset average return over the period T

r_j = jth security/asset average return over the period T

X_j = Portfolio allocation of security/asset j not greater than asset upper limit or global limit µ

P = The minimum return required by a particular investor or trustee of a portfolio

ω = Total asset under management contained in the portfolio.

We note that the validity of this model are in two parts; the expected return is multivariate normally distributed and the investor prefer lower risk with a preference of risk aversion.

If we apply Minimum Absolute Deviation in estimation output (Return on Investment plus initial Asset Under Management). Let us say as follows;

L(A) = {x: x produces A}. where x,A are inputs and output vector respectively. The input level set L(A) satisfies the following properties;

1) $L(0) = R_+^n, 0 \notin L(A)\ for\ A > 0$
2) $x \in L(A), x' \geq x \Rightarrow x' \in L(A)$
3) $A_2 \geq A_1 \geq 0 \Rightarrow L(A_2) \subseteq L(A_1)$

Let us say we input, Equity, Bond, & Money Market returns with expectation of output equal to A (Asset Under Management) L(E,B,M) $\in A$

Let Φ(x), $x \in R_{+}^{n}$ be a frontier optimization function. As an optimization problem Φ(x) may be expressed as,

$\Phi(x) = Max\{A: x, \in L(A)\}\ 0 \leq A < \infty$, $\Phi(x)$ succeeds properties from $L(A)$

(i) Φ(0) = 0, Maximum output produced by a null vector is zero

$\Phi(0) = \text{Max}\{A: 0 \in L(A)\} \Rightarrow 0 \in L(A) \Rightarrow A = 0 \Rightarrow \Phi(0) = 0$

(ii) $x' > x \Rightarrow \Phi(x') \geq \Phi(x),$

$maximum\ output\ produced\ by\ a\ larger\ input\ vector\ is\ lasrger$

(iii) Φ(x) is concave function of x

Input Level

Let

$L_{\Phi}(A) = \{x: F(A, x) \geq 1\}$ Where $F(A, x) = [Min\{\lambda: \lambda\ x \in L(A)\}]^{-1} = \frac{\Phi(x)}{A}$,

then

$$L_{\Phi}(A) = L(A) = \left\{x: \frac{\Phi(x)}{A} \geq 1\right\} = \{x: \Phi(x) \geq A\}.$$

Consider now the Cobb Douglas production frontier given by;

$$\widehat{w}_1 = E \prod_{j=1}^{n} x_{ij}^{j}. \qquad (4)$$

This is the ith decision making unit. Taking logarithm on both sides of (4)

$$\Rightarrow \ln \widehat{w}_i = \ln E + \sum_{j=1}^{n} \propto_j X_{ij} \Rightarrow \hat{y} = a + \sum_{j=1}^{n} X_{ij} \propto_j.$$

If

$$\widehat{W}_i \geq W_i$$

then

$$a + \sum_{j=1}^{n} X_{ij} \propto_i \geq Y_i. \qquad (5)$$

If there are k asset classes making decision of AUM growth (k growth-decision making units), then introducing slack variables s_i , $i = 1,2, \ldots, k$, the inequality is converted equation;

$$\Rightarrow a + \sum_{j=1}^{n} X_{ij} \propto_j - s_i = Y_i \Rightarrow \left[a + \sum_{j=1}^{n} X_{ij} \propto_j - Y_i\right] = s_i.$$

Taking summation on both sides implies

$$\sum_{n=1}^{k} s_i = ka \sum_{i=1}^{k} \sum_{j=1}^{n} X_{ij} \propto_j - \sum_{j=1}^{k} Y_i. \qquad (6)$$

By dividing (6) by k, we have

$$\bar{s} = a + \sum_{j=1}^{n} \bar{X}_j \propto_j - \bar{Y}_i. \qquad (7)$$

Minimizing of (2) is same as minimizing of (7) ($\bar{Y}$being a constant).
Minimization of (5) implies

$$\bar{s} = a + \sum_{j=1}^{n} \bar{X}_j^n \propto_j. \qquad (8)$$

Combining (7) and (8) we obtain a linear programing problem (LPP) for which decision variables are α and α_j as:

$$\text{Min } a + \sum_{j=1}^{n} \bar{X}_j \alpha_j \qquad (9)$$

$$\text{Subject to } a + \sum_{j=1}^{n} X_{ij} \propto_j \geq Y_i$$

$$\alpha_j \geq 0, \text{ is conditional for sign.}$$

Let $a = \alpha^+ - \alpha^-$, then the LPP can be expressed as follows:

$$\text{Minimize } Z = \alpha^+ - \alpha^- + \sum_{j=1}^{n} \bar{X}.j\alpha_j \qquad (10)$$

$$\text{Subject to } \alpha^+ - \alpha^- + \sum_{j=1}^{n} \bar{X}_{ij}\, \alpha_j \geq Y_i$$

$$\alpha^+, \alpha^-, \alpha_j \geq 0, j = 1, 2, 3, \dots, k$$

With two errors "u and v" one sided and the two-sided disturbance term, the Minimum Absolut Deviation (MAD) model is given by,

$$\alpha^+ - \alpha^- + \sum_{j=1}^{n} \bar{X}_j\, \alpha_j = Y_i + u_i + v_i\ , \qquad (11)$$

where $0 \leq u_i < \infty, -\infty < v_i < \infty$.

For ith decision making unit the MAD model implies

$$\alpha^{+} - \alpha^{-} + \sum_{j=1}^{n} \bar{X}_j \, \alpha_j = Y_i + u_i = v_i \, .$$

Taking Modules on both sides, we have

$$\left| a^{+} - a^{-} + \sum_{j=1}^{n} \bar{X}_j \propto_j - Y_i - u_i \right| = |v_i| = v_i^{+},$$

where

$$v_i = v_i^{+} + v_i^{-} \, , v_i^{+} = Max\{0, v_i\}, v_i^{-} = Min\{0, v_i\}.$$

Thus the optimization problem equal to MAD estimated model is given by,

$$\text{Min} \sum_{i=1}^{k} (v_i^{+} + v_i^{-}) \qquad (12)$$

$$\text{Subject to} \quad a^{+} - a^{-} + \sum_{j=1}^{n} X_{ij} \, \alpha_j - u_i - v_i^{+} - v_i^{-} = Y_i$$

$$a^{+}, a^{-}, \alpha_j, u_i, v_i^{+} \; and \; v_i^{-} \geq 0, i = 1,2,3, \dots, m, j - 1,2,3, \dots, n$$

The decision variable of the above linear programming are $\alpha_j, u_i \; and \; v_i^{+} \; and \; v_i^{-}$ the optimal solution of LPP (9) tells DMU specific technical efficiency.

For efficiency estimation, consider the Cobb Douglas production function.

$$w = E \prod_{i=1}^{m} x_i^{\propto j} \, u, 0 \leq \text{u} \leq 1, \qquad (13)$$

and define $u = e^{-z}$; $0 < z < \infty$.

Let the random variable z follow Weibull distribution as in [5] so that;

$$f(x\colon k,\lambda,\theta) = \frac{k}{\lambda}\left(\frac{x-\theta}{\lambda}\right)^{k-1} \exp\left\{-\left(\frac{x-\theta}{\lambda}\right)^{k}\right\}.$$

Let

$$z = \frac{x-\theta}{\lambda} \Rightarrow f(z,k) = \frac{k}{\lambda}(z)^{k-1} \exp\{-(z)^{k}\}.$$

Define $u = e^{-z^{k}} \; or \; u = e^{-y} \, , where \; y = z^{k} \Rightarrow \frac{dy}{dz} = kz^{k-1}$.

$$\ln u = -y \Rightarrow -\ln u = y \Rightarrow \frac{dy}{du} = -\frac{1}{u} \Rightarrow dy = -\frac{du}{u} \Rightarrow y = \ln\left(\frac{1}{u}\right).$$

Notice that

$$y = 0 \Rightarrow u = 1, y = \infty \Rightarrow u = o \ and \ \frac{du}{dz} = \frac{dy}{dz}\frac{du}{dy} = -kz^{k-1}u.$$

Recall that

$$y = z^k \Rightarrow z = y^{\frac{1}{k}} \Rightarrow z^{k-1} = y^{\frac{1}{k}(k-1)} = y^{1-\frac{1}{k}}.$$

Therefore

$$f(y,k) = \frac{k}{\curlywedge} y^{\frac{k-1}{k}} \mathrm{e}^{-\mathrm{y}} = \frac{k}{\curlywedge}\left(\ln\frac{1}{u}\right)^{k-1} du,$$

where $\Gamma\left(\frac{k-1}{k}\right) = \int_0^\infty y^{\frac{k-1}{k}-1} e^{-y} dy.$

The probability density function of u (for $\curlywedge = 1$) is given by,

$$g(u,k) = k\left(\ln\frac{1}{u}\right)^{k-1}. \tag{14}$$

Here k is shape parameter of the distribution $g(u,k)$, k < 1 implies that a greater proportion of DMUs are

efficient, k = 1 implies uniform sufficiency and k > 1 implies that a greater proportion of DMUs are inefficient.

The average level of efficiency of the industry comprising of several DMU is,

$$\bar{u} = E(u) = \int_0^\infty \begin{matrix} \exp(-y)\, ky^{\frac{k-1}{k}} \exp(-y)\, dy \\ = k\int_0^\infty y^{\frac{k-1}{k}} \exp(-2y)\, dy \end{matrix}.$$

Put $2y = v, 2dy = dv \Rightarrow dy = \frac{1}{2}dv \Rightarrow y = \frac{v}{2}$, then

$$\begin{aligned} E(u) &= \frac{k}{2}\int_0^\infty \left(\frac{v}{2}\right)^{\frac{k-1}{k}} \exp(-v)\, dv \\ &= \frac{k2^{\frac{1}{k}}}{4}\int_0^\infty v^{1-\frac{1}{k}} \exp(-v)\, dv \\ &= \frac{k2^{\frac{1}{k}}}{4}\Gamma\left(\frac{k-1}{k}\right). \end{aligned}$$

Note: For $k = 1$, we have $\bar{u} = \frac{1}{2}$, which coincides with $\bar{u}$ as in [6] for $\curlywedge = 1$.

By the Least Square Method the Cobb-Douglas production specification is

$$w_i = E \prod_{j=1}^{m} x_{ij}^{\beta j} u_j, i = 1,2,3 \ldots k$$

$$\ln w_i = \ln E + \sum_{j=1}^{m} \ln x_{ij} + \ln u_i$$

$$\Rightarrow w_i = a + \sum_{j=1}^{m} \beta_j X_{ij} - y_i \tag{15}$$

We have,

$$E(y_i) = k \int_0^{\infty} y_i y_i^{\frac{k-1}{k}} \exp(-y_i)\, dy_i$$

$$= k \int_0^{\infty} y_i^{\frac{k-1}{k}+1} exp(-y_i)\, dy_i$$

$$= k \int_0^{\infty} y_i^{2-\frac{1}{k}} exp(-y_i)\, dy_i$$

$$= k\Gamma(\frac{k-1}{k} + 1).$$

$$E(y_i^2) = k \int_0^{\infty} y_i^{\frac{k-1}{k}+2} exp(-y_i)\, dy_i$$

$$= k\Gamma(\frac{k-1}{k} + 2)$$

$$E(y_i) = E(y_i^2) - [E(y_i)]^2 = k\Gamma(\frac{k-1}{k} + 2) - \left[k\Gamma(\frac{k-1}{k} + 1)\right]^2$$

Let $d = \frac{k-1}{k}$

$$= k\Gamma(d + 2) - [k\Gamma(d + 1)]^2$$

$$= k(d + 1)d\Gamma(d) - k^2 d^2 \Gamma^2(d)$$

$$= kd^2\Gamma(d) + kd\Gamma(d) - k^2 d^2 \Gamma^2(d)$$

$$\leq kd\Gamma(d) = k\left(\frac{k-1}{k}\right)\Gamma\left(\frac{k-1}{k}\right)$$

$$= \Gamma\left(\frac{1}{2}\right),\ iff\ k = 2. \qquad (16)$$

By [7] we write

$$E(y_i) = \frac{n!}{4}\left(n+\frac{1}{2}\right)^{-\left(n+\frac{1}{2}\right)} e^{\left(n+\frac{1}{2}\right)}. \qquad (17)$$

3. Forecast Model & Solution to Contributor Request

From table 3, values are inputted into equation (12) and further which are transformed equations into linear forms. This leverages into the avoidance of large covariance matrix required to solve optimization through Markowitz model [8]. We will therefore, set the pension investment outcome of AES 2013 into its linear form for optimization. Here is setting the request of fund contributors to be that, investment manager grows portfolio by **15%** which is better that earlier growth of less than **12%**. The question is that from growth rates, how the portfolio assets will be allocated to achieve 15% growth rate by end of next period.

Progressively, we shall employ results from simulation of financial derivative to modify our asset class in table 3. This new portfolio will be re-optimized. Our simulation is modeled from forecasting asset prices and setting exercise prices with maturities. Our stock forecast will be through;

$$dV_t^i = V_t^i\left[\left(1-\sum_{i=1}^{n}\theta_t^i\right)\frac{dS_t^0}{S_t^0} + \sum_{i=1}^{n}\theta_t^i\frac{dS_t^i}{S_t^i}\right],\ V_0^0 = v_0\,. \qquad (18)$$

Here

$$\frac{dS_t^0}{S_t^0} = rdt, S_0^0 = s^0, r = r_t \text{ is the risk-free rate,}$$

$$\frac{dS_t^i}{S_t^i} = \mu^i dt + \sum_{i=1}^{k}\sigma^i dW_t^i\,, t \in [0,T],\ S_0^i = s^i. i = 1,\dots,n.$$

The $\mathcal{F}_t$- predictable process $\theta = (\theta_t^0, \theta_t^1, \dots, \theta_t^n)$, is the investment strategy with $\theta_t^i (i = 1, \dots, n)$ denotes the fraction of wealth invested in the risky asset i at time t .

Stock prices are the values of their underlying derivatives. The assets on themselves follow a decay process as the heat diffusion. We therefore view the net value asset prices as the Black-Scholes theory of the case of the multiple assets of the form;

$$S_t^{(i)} = S_0^{(i)} exp\left[\left(\mu^{(i)} - \frac{1}{2}\left[\sigma^{(i)}\right]^2\right) + \sigma W_t^{(i)}\right], t\epsilon[0, T], \tag{19}$$

where $W_t^{(i)}, i = 1, \ldots, d$ are possibly correlated Brownian Motions. We may therefore concatenate to mix in pension portfolio as a product of derivative to modify our return rate. This is possible if regulation includes the buying of derivative product with pension funds. This work is only simulating but is not yet applicable until when PenCom approve the inclusion of financial derivative as part approved investment of pension fund. Nigeria has only Forex Futures as the only existing exchange traded derivative product.

Amount change in the price is equal to the certainty of movement of the price plus the uncertainty caused by the volatility. Therefore, we have;

$$\ln\left[\frac{S_t^{(i)}}{S_{t-1}^{(i)}}\right] = \bar{\mu}^{(i)} + \sigma W_t^{(i)}, \tag{20}$$

where, $\boldsymbol{ln}\left[\frac{\boldsymbol{S_t^{(i)}}}{\boldsymbol{S_{t-1}^{(i)}}}\right]$ are the multiple periodic return and continuously compounded,

$\bar{\mu}^{(i)} = \left(\mu^{(i)} - \frac{1}{2}\left[\sigma^{(i)}\right]^2\right)\tau$ are constant multiple drifts, $\sigma W_t^{(i)}$ multiple random shocks and the expected periodic rate of returns;

$$\mu^{(i)} = \frac{\bar{\mu}^{(i)}}{\tau} + \frac{1}{2}\left[\sigma^{(i)}\right]^2.$$

3.1. Portfolio Equation

General representation of above could be written as,

$$Min(Max) C^T X$$

$$\text{Subject to:} \begin{cases} AX \leq a \\ BX \leq b \\ Lb \leq X \leq ub \end{cases}. \tag{21}$$

Solution to (19) using the interior point method for LP could be associated to the following process;

$Let\ A \in \mathbb{R}^{mxn}, a \in \mathbb{R}^m, B \in \mathbb{R}^{pxn}, b \in \mathbb{R}^p$ and further, the linear programming problem,

$$Min\ C^T X$$

$$\text{Subject to: } \begin{cases} AX \leq a \\ BX \leq b \\ lb \leq X \leq uB \end{cases},$$

where (lb = Lower bound, uB = Upper bound).

We now setting $X_e = X - lb$ to achieve;

$$Min\ (C^T X_e - C^T lb)$$

$$\text{Subject to: } \begin{cases} AX \leq a - A(lb) \\ BX \leq b - B(ib) \\ 0 \leq X_e \leq ub - lb \end{cases}. \tag{22}$$

We can then introduce slack variables S $\in \mathbb{R}^m\ and\ S' \in \mathbb{R}^n$ but can now rewrite the LP as

$$Min\ (C^T X_e - C^T lb)$$

$$\text{Subject to: } \begin{cases} AX + S \leq a - A(lb) \\ BX = b - B(lb) \\ X_e + S' = ub - lb \\ X_e \geq 0, S \geq 0, S' \geq 0 \end{cases}.$$

Using single matrix representation gives [8];

$$Min\ (C^T X_e - C^T lb)$$

$$\text{Subject to:} \begin{pmatrix} A & I_m & 0_{mxn} \\ B & 0_{pxm} & 0_{pxn} \\ I_n & 0_{nxm} & I_n \end{pmatrix} \begin{pmatrix} X_e \\ S \\ S' \end{pmatrix} = \begin{pmatrix} a - A(ib) \\ b - B(lb) \\ ub - lb \end{pmatrix}. \tag{23}$$

$$X_e \geq 0, S \geq 0, S' \geq 0$$

This, without a problem from constants of objective function, we have the dual of the LP as;

$$\max b^T w$$

$$\text{Subject to;} \begin{cases} A^T w + S^{'} \leq C \\ w \in \mathbb{R}^m \end{cases} . \tag{24}$$

Introducing a slack $S'' \in \mathbb{R}^m$, as in minimization function, we have;

$$\max b^T w$$

$$\text{Subject to} \begin{cases} A^T w + S^{'} \leq C \\ w \in \mathbb{R}^m, S^{'} \geq 0 \end{cases} . \tag{25}$$

From both dual and original LP, we arrive at optimal solution; Noting that a vector (X^*, w^*, S^*) is a solution of the primal-dual if and only if it satisfies the Karush-Kuhn Tucker (KKT) optimality condition. The KKT condition here can be written as;

$$\begin{cases} A^T w + S^{'} = CX \\ X_i, S_i^{'} = 0, i = 0,1, \dots, n \end{cases} \tag{26}$$

Minimum Absolute Deviation (MAD) Mathematical model

In this formulation, we are following Konno and Yamazaki method of minimizing risk through the minimization of the absolute deviation from our table 3 returns of the assets in our portfolio problem. Our mathematical steps to provide solution is as follows;

(a) Calculation of absolute deviation of each asset
(b) Linearization of the objective function from original classical Markowitz model
(c) Define the Y_t variables and described as a linear mapping of the non-linear variance model, that is; $|\sum_{i=1}^{n}(x_{it} - \bar{r}_i)x_i|$
(d) We define the five (5) assets as our variables including, x_1, x_2,,x_3, x_4, & x_5.
(e) Setting the objective function in a linear form as, $\min z = \frac{1}{n}(y_1 y_2 + \cdots + y_n)$, then we our constraints as follows,
    (i) Budget constraint, $\sum_{i=1}^{n} v_i = A$
    (ii) The return demand constraint, $\sum_{i=1}^{n} r_i v_i \geq \rho A$

(f) Next we take time to relate the objective function variables to y_t to variables used in the other constraints; v_i, v_2, … v_n

In consideration of the relation and starting, from 1st year, that is setting the relation as,

$$y_t = \left|\sum_{i=1}^{n}(x_{it} - \bar{r})x_i\right| \mathit{in\ this\ case,}$$

$$y_1 \dots y_n = y_1, y_2, y_3, y_4, y_5$$

Therefore $y_t = -\sum_{i=1}^{5}(x_i - \bar{r})x_i \; OR + \sum_{i=1}^{5}(x_{it} - \bar{r})x_i$

Table 4: Absolute Deviation of each asset covering 5-year period.

	V(1)	V(2)	V(3)	V(4)	V(5)
t(1)	-0.0084	-0.0112	0.0016	-0.0122	0.0036
t(2)	-0.0054	-0.0077	-0.0094	-0.0082	-0.0104
t(3)	0.0036	0.0053	0.0016	0.0048	0.0006
t(4)	0.0046	0.0063	0.0026	0.0078	0.0026
t(5)	0.0056	0.0073	0.0036	0.0078	0.0036
MS Excel Avg Deviation	0.00552	0.00756	0.00376	0.00816	0.00416

From Table 4, we realise our y_t and thus our objective function;

y_1 -0.0084v_1 -0.0112v_2 + 0.0016v_3 - 0.0122v_4 + 0.0036$v_5 \geq 0$

y_1 +0.0084v_1 + 0.0112v_2 - 0.0016v_3 + 0.0122v_4 - 0.0036$v_5 \geq 0$

y_2 -0.0054v_1 -0.0077v_2 -0.0094v_3 -0082v_4 -0.0104$v_5 \geq 0$

y_2 +0.0054v_1 + 0.0077v_2 -0.0094v_3 + 0082v_4 + 0.0104$v_5 \geq 0$

y_3 +0.0036v_1 + 0.0053v_2 + 0.0016v_3 + 0.0048v_4 + 0.0006$v_5 \geq 0$

y_3 -0.0036v_1 - 0.0053v_2 - 0.0016v_3 - 0.0048v_4 - 0.0006$v_5 \geq 0$

$y_4 + 0.0046v_1 + 0.0063v_2 + 0.0026v_3 + 0.0078v_4 + 0.0026v_5 \geq 0$

$y_4 - 0.0046v_1 - 0.0063v_2 - 0.0026v_3 - 0.0078v_4 - 0.0026v_5 \geq 0$

$y_5 + 0.0056v_1 + 0.0073v_2 + 0.0036v_3 + 0.0078v_4 + 0.0036v_5 \geq 0$

$y_5 + 0.0056v_1 + 0.0073v_2 + 0.0036v_3 + 0.0078v_4 + 0.0036v_5 \geq 0$

We then set our objective function as a linear derivation from the classical Makowitz model as thus;

Min $f(z) = z; 0.00552v_1 + 0.00756v_2 + 0.00376v_3 + 0.00816v_4 + 0.00416v_5$

Subject to;

(a) The budget constraint;

$v_1 + v_2 + v_3 + v_4 + v_5 = 432.688$

(In this case of above constraint it is formulated as equality constraint)

(b) We add the return demand constraint. We note that our return demand constraint is mimicking our original return demand to maintain a pattern of same client contributor request.

$0.0984v_1 + 0.1357v_2 + 0.1164v_3 + 0.1422v_4 + 0.1324v_5 \geq 64.90$

(c) Finally, we add our limitation constraint on assets classes,

(i) $21.63 \leq v_1 \leq 108.172$

(ii) $0 \leq v_2 \leq 151.44$

(iii) $0 \leq v_3 \leq 346.15$

(iv) $0 \leq v_4 \leq 86.54$

(v) $0 \leq v_5 \leq 86.54$

where;

X_1 is the value of exposure of the portfolio to equity stocks

X_2 is the value of exposure of the portfolio to Money Market Securities

X_3 is the value of exposure of the portfolio to FGN bond

X_4 is the value of exposure of the portfolio to State bonds

X_5 is the value of exposure of the portfolio to Corporate bond

$\sum_{i=1}^{5} X_i = X.$

Modification of Portfolio with Financial Derivative from Stock price forecasting Using Monte Carlo Method.

We recall our mathematical model (18) – (20) which is a solution of forecasting stock prices relevant in simulating derivative of financial assets. Procedures to generate data for simulating derivative are listed below (see table 5);

Table 5: Forecast of Bank stock prices for September 2015 – November 2015

Mean	0.000258	Variance	0.118268	Std	0.338485	Drift	-0.05888
Date	Price	Price Log Return	N(0,1)	Log Return + Shock	S(T+)	Forecast Date	
Jul-28	4.99		5.03				
Aug-03	4.87	-0.02434	-0.16329	-0.11415	4.487397	Sep-29	
Aug-04	4.87	0	-0.05992	-0.07916	4.145878	Sep-30	
Aug-05	4.87	0	1.025239	0.288151	5.530431	Oct-02	
Aug-06	4.81	-0.0124	0.680091	0.171324	6.563932	Oct-05	
Aug-07	4.77	-0.00835	1.38771	0.410842	9.898983	Oct-05	
Aug-10	4.76	-0.0021	0.024143	-0.0507	9.409575	Oct-06	
Aug-11	4.65	-0.02338	-1.09049	-0.42799	6.133326	Oct-07	
Aug-12	4.58	-0.01517	-0.81024	-0.33313	4.395621	Oct-08	
Aug-13	4.42	-0.03556	-0.02415	-0.06705	4.110551	Oct-09	
Aug-14	4.35	-0.01596	1.477626	0.441277	6.390639	Oct-12	
Aug-17	4.1	-0.05919	0.340342	0.056324	6.760917	Oct-13	
Aug-18	4.2	0.024098	2.417853	0.75953	14.44991	Oct-14	
Aug-19	4.07	-0.03144	-0.51606	-0.23355	11.44022	Oct-15	
Aug-20	4.27	0.047971	0.316215	0.048158	12.00463	Oct-16	
Aug-21	4.69	0.093819	0.631052	0.154725	14.01345	Oct-19	

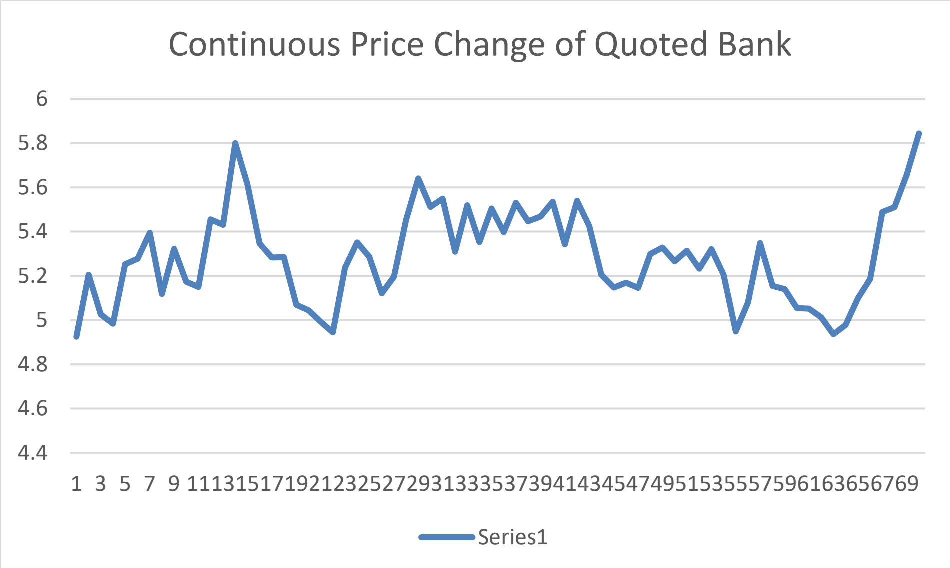
Aug-31	5.14	0.09162	-1.11034	-0.43471	9.073035	Oct-20	
Sep-01	5.3	0.030654	0.373878	0.067676	9.708313	Oct-21	
Sep-02	5.29	-0.00189	-0.34279	-0.1749	8.150489	Oct-22	
Sep-03	5.02	-0.05239	0.138605	-0.01196	8.053586	Oct-23	
Sep-04	5.1	0.015811	-1.71675	-0.63997	4.24673	Oct-26	
Sep-08	5.09	-0.00196	0.242204	0.023106	4.345997	Oct-27	
Sep-09	4.9	-0.03804	-0.80818	-0.33243	3.116848	Oct-28	
Sep-10	4.9	0	0.134248	-0.01344	3.075252	Oct-29	
Sep-11	4.82	-0.01646	2.024047	0.626232	5.752411	Oct-30	
Sep-15	5.02	0.040656	1.068471	0.302785	7.786597	Nov-02	
Sep-16	5.05	0.005958	-1.29062	-0.49573	4.743017	Nov-03	
Sep-17	5.05	0	0.740846	0.191889	5.746334	Nov-04	
Sep-18	5.18	0.025417	0.085843	-0.02982	5.577511	Nov-04	
Sep-21	5.18	0	0.492652	0.107879	6.21286	Nov-05	
Sep-22	5.23	0.009606	1.41064	0.418604	9.44254	Nov-09	
Sep-28	5.03	-0.03899	-0.69943	-0.29562	7.025899	Nov-10	

1. Collate periodic closing stock prices
2. Compute the mean, variance and standard deviation
3. Compute the periodic return. This generates our log returns
4. Generate set of random numbers
5. Forecast price using (19)
6. Use closing price and forecast price to create derivative.
7. Evaluate return and include in portfolio to stimulate different option (see figure 1).

Table 6: Simulating Option Parameters from Table 5,

S/N		July 28	Aug 06	Aug 13	Aug 20	Aug 31
	Exercise Price	5.49	5.29	4.86	4.70	5.65
	Premium	0.549	0.529	0.486	0.470	0.565
	Maturity - Days	180	90	120	30	210
	Volatility	33%	33%	33%	33%	33%
	Risk free rate	11.55%	12%	12.5%	11.76%	12.25%
	Current Price	6.65	6.80	9.15	4.42	5.90

Figure 1: Stock Price movement graph by Monte-Carlo formula



3.3 Simulation of Options for Mixed Portfolio with Derivative for Optimization

The Option contract in our introduction was explained to be "Call Option" and "Put Option". A call option is purchased at exercise price, that is; a 10% of total of the exercise price is paid as value of buying the right to buy at the exercise price of the underlying asset on or before a given expiration date. This contract gives the writer the obligation to sell but gives the buyer the right to buy but not the obligation to buy. It is a common practice to exercise call option if a market price at any time on or before the expiration date is greater than the exercise price. On expiration, if the value of the underlying stock exceeds the exercise price, then; the value of the call is unattractive to exercise. To build simulation formulation we will use the following notations;

1. In the money-call = $S_T \geq X$
2. Expired unexercised call = $S_T \leq X$
3. Net Value = Option value - Premium

We will create options using the following criteria for this study (see tables 8-10);

i. Exercise price = $1.1X_0 = X_t$
ii. Premium = $0.1X_t$

iii. Option Value = $X_T - X_0$

iv. X_T = $\frac{1}{6}\sum_{t=1}^{6} x_t$

Table 7: Result of Option values from Black-Scholes Formula Using MS-Excel

Black-Scholes Option Pricing Model								
Jul-28			**Aug-06**			**Aug-13**		
Time to Expiration		0.49315068	Time to Expiration		0.24657534	Time to Expiration		0.32876712
Exercise Price		NGN 5.49	Exercise Price		NGN 5.29	Exercise P	4.86	NGN 4.86
Current Stock Price		NGN 6.65	Current Stock Price		NGN 6.80	Current Stock Price		NGN 9.15
Volatility		33.00%	Volatility		33.00%	Volatility		33.00%
Risk-Free Rate		11.55%	Risk-Free Rate		12.00%	Risk-Free Rate		12.50%
d1		1.18882286	d1		1.79487811	d1		3.65567535
d2		0.95708139	d2		1.63101214	d2		3.46645923
N(d1)		0.88274531	N(d1)		0.96366344	N(d1)		0.99987175
N(d2)		0.83073691	N(d2)		0.94855612	N(d2)		0.99973632
Call Option Value		NGN 1.56	Call Option Value		NGN 1.68	Call Option Value		NGN 4.49
Intrinsic Value		NGN 1.16	Intrinsic Value		NGN 1.51	Intrinsic Value		NGN 4.29
Speculative Prem.		NGN 0.40	Speculative Prem.		NGN 0.17	Speculative Prem.		NGN 0.20
Put Option Value		NGN 0.10	Put Option Value		NGN 0.02	Put Option Value		NGN 0.00
Intrinsic Value		NGN -	Intrinsic Value		NGN -	Intrinsic Value		NGN -
Speculative Prem.		NGN 0.10	Speculative Prem.		NGN 0.02	Speculative Prem.		NGN 0.00
	Call			**Call**			**Call**	
	Delta	0.8827		Delta	0.9637		Delta	0.9999
	Gamma	0.1277		Gamma	0.0715		Gamma	0.0003
	Theta	-0.8051		Theta	-0.7646		Theta	-0.5842
	Vega	0.9190		Vega	0.2691		Vega	0.0026
	Rho	2.12460674		Rho	1.20120738		Rho	1.53307147
	Put			**Put**			**Put**	
	Delta	-0.1173		Delta	-0.0363		Delta	-0.0001
	Gamma	0.1277		Gamma	0.0715		Gamma	0.0003
	Theta	-0.2061		Theta	-0.1483		Theta	-0.0012
	Vega	0.9190		Vega	0.2691		Vega	0.0026
	Rho	-0.43288976		Rho	-0.06514614		Rho	-0.00040435

Table 8: Continuation of Table 7

Black-Scholes Option Pricing Model						
Aug-20				Aug-31		
Time to Expiration		0.082191781		Time to Expiration		0.575342466
Exercise Price		NGN 4.70		Exercise Price		NGN 5.65
Current Stock Price		NGN 4.42		Current Stock Price		NGN 5.90
Volatility		33.00%		Volatility		33.00%
Risk-Free Rate		11.75%		Risk-Free Rate		12.25%
d1		-0.499851029		d1		0.579697193
d2		-0.594459089		d2		0.329387795
N(d1)		0.308589988		N(d1)		0.718940582
N(d2)		0.276102549		N(d2)		0.629068704
Call Option Value		NGN 0.08		Call Option Value		NGN 0.93
Intrinsic Value		NGN -		Intrinsic Value		NGN 0.25
Speculative Prem.		NGN 0.08		Speculative Prem.		NGN 0.68
Put Option Value		NGN 0.31		Put Option Value		NGN 0.29
Intrinsic Value		NGN 0.28		Intrinsic Value		NGN -
Speculative Prem.		NGN 0.03		Speculative Prem.		NGN 0.29
	Call				**Call**	
	Delta	0.3086			Delta	0.7189
	Gamma	0.8420			Gamma	0.2284
	Theta	-1.0467			Theta	-0.8386
	Vega	0.4462			Vega	1.5092
	Rho	0.10563369			Rho	1.905742068
	Put				**Put**	
	Delta	-0.6914			Delta	-0.2811
	Gamma	0.8420			Gamma	0.2284
	Theta	-0.4997			Theta	-0.1936
	Vega	0.4462			Vega	1.5092
	Rho	-0.276954918			Rho	-1.123723641

Table 9: Extracting Call Option Net-Values and Rate of Return

S/No	Option	Option Value	Option Premium	Net Value	Rate of Return
	July 28	1.56	0.549	1.011	0.184
	August 06	1.68	0.529	1.151	0.218
	August 13	4.49	0.489	4.004	0.823
	August 20	0.08	0.470	-0.39	-0.08
	August 31	0.93	0.565	0.365	0.06
Average					**0.241**

Table 10: Absolute Deviation with simulated Derivative Call-Option

	V(1)	V(2)	V(3)	V(4)	V(5)	V(6)
Y(1)	-0.0084	-0.0112	0.0016	-0.0122	0.0036	-0.0572
Y(2)	-0.0054	-0.0077	-0.0094	-0.0082	-0.0104	-0.0232
Y(3)	0.0036	0.0053	0.0016	0.0048	0.0006	0.5828
Y(4)	0.0046	0.0063	0.0026	0.0078	0.0026	-0.3212
Y(5)	0.0056	0.0073	0.0036	0.0078	0.0036	-0.1812
MS Excel Avg Deviation	0.00552	0.00756	0.00376	0.00816	0.00416	0.23312

From Table 10, we realize our y_t with the inclusion of Call-Option return which calibrates our new constraints and objective function;

y_1 $-0.0084v_1$ $-0.0112v_2 + 0.0016v_3 - 0.0122v_4 + 0.0036v_5 – 0.0572v_6 \geq 0$

y_1 $+0.0084v_1 + 0.0112v_2 - 0.0016v_3 + 0.0122v_4 - 0.0036v_5 + 0.052v_6 \geq 0$

y_2 $-0.0054v_1$ $-0.0077v_2$ $-0.0094v_3$ $-0082v_4$ $-0.0104v_5 – 0.0232v_6 \geq 0$

y_2 $+0.0054v_1 + 0.0077v_2$ $-0.0094v_3 + 0082v_4 + 0.0104v_5 + 0.0232v_6 \geq 0$

y_3 $+0.0036v_1 + 0.0053v_2 + 0.0016v_3 + 0.0048v_4 + 0.0006v_5 + 0.586v_6 \geq 0$

y_3 $-0.0036v_1 - 0.0053v_2 - 0.0016v_3 - 0.0048v_4 - 0.0006v_5 – 0.586v_6 \geq 0$

y_4 $+0.0046v_1 + 0.0063v_2 + 0.0026v_3 + 0.0078v_4 + 0.0026v_5 – 0.321v_6 \geq 0$

y_4 $-0.0046v_1 - 0.0063v_2 - 0.0026v_3 - 0.0078v_4 - 0.0026v_5 + 0.321v_6 \geq 0$

y_5 $+0.0056v_1 + 0.0073v_2 + 0.0036v_3 + 0.0078v_4 + 0.0036v_5$ $-0.181v_6 \geq 0$

y_5 $-0.0056v_1 - 0.0073v_2 - 0.0036v_3 - 0.0078v_4 - 0.0036v_5 - 0.181v_6 \geq 0$

We then re-set our objective function as a linear derivation from the classical Makowitz model as thus;

Min $f(z) = z;\ 0.00552v_1 + 0.00756v_2 + 0.00376v_3 + 0.00816v_4 + 0.00416v_5 + 0.233v_6$

Subject to;

(d) The budget constraint;

$$v_1 + v_2 + v_3 + v_4 + v_5 + v_6 = 432.688$$

(In this case of above constraint it is formulated as equality constraint)

(e) We add the return demand constraint. We note that our return demand constraint is mimicking our original return demand to maintain a pattern of same client contributor request.

$$0.0984v_1 + 0.1357v_2 + 0.1164v_3 + 0.1422v_4 + 0.1324v_5 + 0.241v_6 \geq 64.90$$

(f) Finally, we add our limitation constraint on assets classes,

(vi) $21.63 \leq v_1 + v_6 \leq 108.172$

(vii) $0 \leq v_2 \leq 151.44$

(viii) $0 \leq v_3 \leq 346.15$

(ix) $0 \leq v_4 \leq 86.54$

(x) $0 \leq v_5 \leq 86.54$

4.1. Analysis of Result

Our analysis using MAD model for plain portfolio without simulated call option returns offers us result from the point of view of minimizing our average absolute deviations. This has contained elements of Markowitz classical mean variance model used to represent portfolio risk. Our constraints also contained contributor minimum return demand which deals with portfolio downside risk. This representation of all diversifiable (systematic) risk applauds the use of MAD model for both simplicity and reliability. Tora adjusted resource allocation to portfolio assets assuring the allocation of 20.0% to corporate bond irrespective of fund manager’s allocation of near zero (0) in the actual AES portfolio. Other adjustments include a reduction of FGN bond by 20%, there are no reductions from Money market and State Bond. This optimization resulted to overall return of 13.06% which unfortunately does not meet our contributor minimum return demand of 15%. A further investigation of the optimization result reveals that there is possibility of a maximum return of N847.15m which translates to a possible maximum return rate of 95.78%.

Calibrating our portfolio with simulated call options returns and with minimum return demand improved our asset allocation and better rate of return. The inclusion of options returns from our stock price forecast and simulation of options pricing showed a promise of improved addition of variable income securities through its risk-return rule considerations. This consideration allowed for maximum allowable investment to be used. The reduction of state bond in the mixed portfolio is expected due to high volatility in state owned bonds from historical return in the MAD formulation. Corporate bond was considered to have a better risk-return rate than equity and money market. Equity retained its holding of 5% while Money Market lost its holding of 35% to FGN bond with a gain of 9.45%, Corporate bond with a gain of 20% and Derivative with a gain of 25% full allowable weight for stock in pension regulated portfolio.

In conclusion, we shall be subjecting results from mathematical formulations to inferential test. Returns from optimizations results from different models are grouped for mean comparison with AES manager's returns in Table 1.3. A t-test statistic for a mean comparison of two sets of returns from assets in portfolio to test the variances between groups as a ratio of variance within groups, that is; $= \frac{Variance\ between\ groups}{variance\ within\ groups}$. Our P-Value as a measure of the strength of an evidence against our null hypothesis;

$$H_o: A\ higher\ return\ is\ not\ achievable\ from\ optimizing\ AUM\ in\ Table\ 1.$$

$$H_1: A\ higher\ return\ is\ achievable\ from\ optimizing\ AUM\ in\ Table\ 1.3$$

This is read from P-Value source "socscistatistics.com/effectsize/default3.aspx" using t-value calculated from the formula;

$$t_{cal} = \frac{\bar{X}_{opt} - \bar{X}_{AES}}{S_p * \sqrt{\frac{1}{n_{opt}} + \frac{1}{n_{AES}}}} \tag{27}$$

Where;

$$S_p^2 = \frac{(n_{opt}-1)S_{opt}^2 + (n_{AES}-1)S_{AES}^2}{(n_{opt}-1)(n_{AES}-1)}.$$

Specifically, and by general mathematical standard, we set P-Value as a probability that the pattern of our return results could be further replicated by random data futuristically. Our confidence level is set at 95% with α = 0.05. Thus;

(1) Accept

$$H_o\text{: } There\ is\ no\ difference\ in\ returns\ of\ AES\ and\ optimized\ portfolio$$

if P ≥ 0.05; that is there is a 5% and greater value there is no real difference in AES returns and the returns from optimized AES portfolio.

(2) Reject

$$H_o\text{: } There\ is\ no\ difference\ in\ returns\ of\ AES\ and\ optimized\ portfolio$$

and accept

$$H_1\text{: } There\ is\ difference\ between\ returns\ of\ AES\ and\ optimized\ portfolio$$

if P < 0.05; that is there is 1-0.05 confidence that our results is significantly different and confirms a possibility of achieving higher returns from AES portfolio through our mathematical models in this research work.

Table 12: t-statistic from formula 27 for Research Hypothesis

AES Fund Manager's Returns	**Optimization Returns**			
0.09	0.132211961	Mean	0.11086	0.142293
0.12	0.1486	Variance	0.000232	9.93E-05
0.1043	0.130613849	n	5	5
0.13	0.150041485	n-1	4	4
0.11	0.15	Sp^2	8.29E-05	0.009104
(1/n(1))	0.2			
(1/n(2))	0.2			
Sqr(1/n(1)+1/n(2))	0.632455532	**t**	**5.459**	

From table 12, an independent sample t-test was used to check the effectiveness of optimizing AES portfolio with a contributor specified minimum demand rate-of-return as a stimulator to achieving higher rate of return than AES fund manager's returns, t(4) = 5.459 from manual calculation and t(4) = 7.5 from SPSS software calculation were calculated. P = 0.005473 and P = 0.00169 were the read probabilities from t = 5.459 and t = 7.5 respectively, at 4-df and α = 0.05. With proven significant difference in mean comparison of AES returns and optimization models returns, we therefore reject,

$$H_o: There\ is\ no\ difference\ in\ returns\ of\ AES\ and\ optimized\ portfolio$$

and accept

$$H_1: There\ is\ difference\ between\ returns\ of\ AES\ and\ optimized\ portfolio$$

4.2. Assumptions

Here we assumed that all economic growth indicators are kept constant from opening value to closing value period. This is because we are using retrospective rate of return for a corrective forecast of would have been a better output using "Contributor required Rate of Return" to build portfolio asset allocation. This work is also using investment strategy of "Minimize risk for a specified return" This suits investors whose risk-return appetite is known.

5. Conclusion & Recommendation

We had expectation to challenge pension fund managers to wake to the challenge of investigating more channels of improving contributors return. We realized our expectation in this study. We could see that there is a possibility to allocate funds across the asset classes within their individual regulatory ratios to AUM and achieve 13.06% return instead of 11% achieved by AES fund manager. We further simulated a possible regime of having call options from stocks as underlying security which when added to the AES portfolio promised a 15% return. This is far above 11% and also met our required contributor minimum return demand. We then can recommend that contributors should be empowered to request a minimum rate of return at least twice a year. Pension Fund Administrators should be regulated to take only a percent achievement multiplied by 1.6% asset management fees rate as income for managing pension fund.

We have used a one-year return data of an AES portfolio and a 42 days of stock market trading to arrive at our decisions. We therefore advice that our results be used within the boundaries of available validity of such data, its spread, mathematical model validity and their inherent values with considerations of possible deficiencies in this work. We are recommending that Nigeria’s Pension Commission begin a study on how pension contributors are to be empowered with instruments that can allow them request a minimum return within approved periods. There is also need to approve PFAs asset-management-fees based on their achievement of these periodically set contributor-minimum-return. This clearly defines and regulates pension managers from earning asset management fees even when they practically either did not satisfy 50% of contributors’ expectation in period return.

Conflict of Interests

The author declares that there is no conflict of interests.

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