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OPTIMAL INVESTMENT STRATEGY WITH DEBT FINANCING UNDER STOCHASTIC INTEREST RATES

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Abstract. We study an approach that can be applied by firms' managers in order to make more effective decisions on investment and debt strategies with the consideration of fluctuating interest rates. We model by a stochastic differential equation in which the drift varies in response to expanded investment and outstanding debt. In the optimization of the investment-debt policy, we consider a continuous stochastic interest rate as a discounting factor. One of our findings is that the optimal condition to consider debt financing for investment is when the liquidity level and the liquidity drift are high while the interest rates are low. Also, there are two limiting points over the liquidity level domain whereby below the lower point and above the upper point it is not optimal to consider debt financing for investment. Over the liquidity-drift range we find a point above which debt financing is optimal and also having an interest rate below the threshold value is optimal for debt financing. The suggested approach is more suitable for managing growing firms in the developing economies where the macroeconomic effects are more intense and firms rely more on debt financing over equity financing. Firms' managers have to consider the liquidity level, liquidity drift and the interest rates before embarking in debt financing for investment, while the monetary policy makers in developing economies have to ensure low interest rates in order to favour firms' investment growth.

Keywords: firm investment; debt financing; interest rate; developing economies; stochastic optimal control.

2010 AMS Subject Classification: 93E20.

1. Introduction

There is a good number of models that have been developed in addressing the optimal policies in the management of firms' financing and investment. The literature on such models records some prominent studies by Merton [1], Akian et al. [2], Décamps and Villeneuve [3], and Bolton et al. [4]. In fact any long-term rational investor wants to hedge stochastic changes in investment opportunities, such as changes in interest rate, excess returns and inflation rates [5]. Such investors or firms will adapt to fluctuations in financing conditions by postponing or bringing forward investments, timing favourable market conditions to raise more funds or hedging against unfavourable market conditions [4]. It is stipulated by Eregha [6] that fluctuation in the interest rate has significant effects on business decisions about how to serve and invest. Actually, finding out how capital investment policies of firms are affected by the raise in interest rate is very useful for financial and monetary policies [7].

Amongst the major challenges faced by firms' management is how to rise fund for furthering investment and for carrying out the necessary usual operations at the time of need. During such needy conditions firms may either rise fund from the equity market or move to issue debt from the credit market. Firms decision on whether to consider equity or debt is very critical since the value of a firm depends on its debt-equity ratio and the macroeconomic environment [2, 8, 9]. Agarwal and Mohtadi [10] argue that, significant differences exist between the developed and developing economies in regard of such decisions. From the Pecking-Order point of view as suggested by Myers and Majluf [11], firms' managers prefer retained earnings or debt over equity. Therefore, since the stock markets of developing countries are still inefficient and immature, firm dependence on debt financing is very high whenever there is growth option [12, 13]. Moreover, from the contemporary market-timing-theory, firms issue debt when the interest rates on the debt are low [8]. Thus, much as firms in the developing country economy

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rely mostly on debt financing and therefore supposed to consider the interest rates, an efficient decisions making approach is required.

This study draws attention to the problem of firm's management that wants to optimize the use of retained earnings and debt in order to further its investment in response to fluctuations in the interest rates. We, therefore, find a suitable approach that can be applied by firms' financial managers for satisfying their endeavor to boost up the investment through earnings and debt. The retained earning in this case refers to the accumulated cash from the profit made that has raised the liquidity level of the firm. Though there is a good number of models on optimal policies for firms' financing and investment, unfortunately our survey in the literature revealed that there misses a mathematical model that attends to earning and debt investment financing under stochastic interest rates. The knowledge gap that we aim to fix is to consider the stochastic interest rate in the optimization of the investment side of the model put forward in [3]. In addition, we include debt variable and its effect on the drift of liquidity on the model.

Several studies have considered firm investment strategies supported by equity financing as the only external source of fund [3, 14, 15]. The study by Bolton et al. [14], for example, proposed a mathematical model of investment, financing and risk management for financially constrained firm. One of their findings was that the optimal external financing and payout are characterized by endogenous double-barrier policy for the firm's cash-capital ratio. However the fact that exogenous forces such as the fluctuations in the market interest rate were not explicitly addressed. In this study we take care of the firm's exogenous forces represented by the fluctuations in the interest rate while considering internally generated revenue and debt as sources for funding firm investments.

Recently, there are some studies which have considered debt as the main source of fund in boosting the firms' investments [16, 17, 18, 19, 20]. In all these papers the interest rates have been assumed to be constant, a fact that may not capture well the reality particularly in developing economies. Pierre et al. [17] restructured the performance-sensitive debt model of Manso et al. [21] and proposed a collateralized debt model for capital investment. Their major assumption was that the firm could go bankrupt only when it is unable to collateralize loan with its assets. In one of their findings they recommended that managers should not use all the cash into investment when the book value of equity is low. In addition, they showed that shareholders do abandon highly indebted firms which happens when the firm profitability is less than the cost of debt. On the other hand Sundaresan et al. [20], building on the idea by Myers [22] that firms are collections of growth options and assets at hand, developed a dynamic contingent-claim framework model. In their modelling they considered shocks in demand of the firm's products and constant interest rate. They concluded that the firm's ability to borrow against its asset and the growth options have substantial influence on the investment strategies and its value. Far as we find these results important, we still see that there is a need to move on and provide a means of decision making that caters for the more efficient strategy which will consider the macroeconomic effects on debt financing for the investment. The investment strategy that considers macroeconomic influence in firms' investment and growth is very useful for the growing firms mostly in developing economies where growth options are commonly available.

The fact that the growing firms' performance in developing economies are challenged by technological and managerial innovativeness [23, 24], has triggered our inspiration in undertaking this study. Actually, this appeals for additional new approaches in managing firms towards the desired higher performance. Also, we are motivated by the truth that the low efficiency of capital market does not favour the needs of companies in the developing countries. As a result firms have to resort to loans when they face financial constrains meanwhile being subjected to high and fluctuating interest rates. Bearing in mind that the macroeconomic effects on investment and firm financing in developing economies are more intense than in the developed world [25, 26], there is a need to have ways for firms' performance and survival.

In this study, firm investment and debt levels are modeled in relation to the liquidity value of a firm because we assume that the decision to take loan and/or invest is based on the liquid asset of the firm. In addition, the same decision is passed by after considering the state of market interest rate which in this case is said to be a continuous stochastic process, generally modeled as a term structure [27, 28]. Therefore, the discounting factor applied in the optimization problem of investment-debt levels is the continuous stochastic process as given in [29].

The rest of this paper is organized as follows. Section 2 provides for model formulation, mathematical representation of investment and debt dynamics, bankrupt condition and definition of the objective function. In section 3, we prove the existence and uniqueness of the solution of the value function and solve for the solution by the method of Fourier transform technique. Section 4 is about numerical experiments and discussion. We present 3D plots that show an outlook of the value function over liquidity level, liquidity drift and interest rate in pairs. We also carry out numerical determination of decision making points meanwhile making discussion of the results. In section 5, we make a summary of the results of this study, draw a conclusion and suggest the way forward.

2. Model formulation

We consider a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and a filtration $(\mathcal{F}_t)_{t\geq 0}$ in describing uncertainty. We also refer to B_t as the standard Brownian motion with respect to the filtration $(\mathcal{F}_t)$. The company under consideration runs activities which generate cash that alters its liquidity value Y_t. The liquid value of a company is modeled as a SDE with a drift parameter μ and volatility parameter σ as follows

$$dY_t = \mu dt + \sigma dB_t. \tag{1}$$

We then consider the interest rate r_t that varies randomly between the highest likely interest rate and the lowest likely interest rate. The interest rate r_t as used here indicates the fluctuation in the economy under which the company operates. The interest rate is assumed to be a continuous stochastic process. We denote by I_t the cumulative amount of cash used for investment up to time t and by D_t the total amount of debt accessed up to time t. The processes I_t and D_t are assumed to be non-decreasing, $(\mathcal{F}_t)$-adapted and have sample paths which are left continuous with right limits. The function value $\phi(I_t)$ gives an additive value in the drift μ due to return from investment I_t. Meanwhile the function value $\psi(D_t)$ gives the rate of decrease in the drift μ due to paying back the debt. In fact ϕ and ψ are increasing functions in I_t and D_t respectively. The model for the liquid value dynamics of the company is given by

$$dY_t = [\mu + \phi(I_t) - \psi(D_t)]dt + \sigma dB_t - dI_t + dD_t,\ Y_0 = y. \tag{2}$$

The investment I_t at time t is done only when $(Y_t + D_t) > Y_I$ for a fixed level of possible investment Y_I. We assume that any amount above Y_I can be used for investment, i.e. $I_t = (Y_t + D_t) - Y_I$. The investment can also be done only when the interest rate $r_t < r_\theta$ for the fixed threshold interest rate for investment r_θ. Debt on the other hand can be taken up to the amount αY_t for $0 \leq \alpha < 1$ and it is only exercised when $Y_L < Y_t < Y_I$ where Y_L is the lowest liquidity level for accessing loan. When debt is to finance investment then the total investment should not exceed $Y_I + E$ for a fixed E, beyond that the investment is too risky to be financed through debt.

The conceptual mathematical representations of the investment level I_t and the debt level D_t at time t are respectively given by

$$I_t \mathbb{1}_{(r_t < r_\theta)} = (Y_t + D_t - Y_I)^+ , \tag{3}$$

$$D_t \mathbb{1}_{(Y_L < Y_t < Y_I, I_t \leq Y_I + E)} = \alpha Y_t . \tag{4}$$

We assume that the company should maintain positive liquidity level in order to continue with the business otherwise it undergoes bankruptcy. We therefore define the company time for bankruptcy by

$$\mathsf{T} = \inf\{t \geq 0 : Y_t \leq 0\}. \tag{5}$$

Actually, our objective is to study the optimal investment strategy of a company with the possibility of accessing loan under the fluctuation of the interest rates. The company has the option to access loans under long term debt plans in order to boost investment. Moreover, the company needs to raise its investment as high as possible but is supposed to consider the interest rates. High interest rates have negative effects over investment. The decisions on investment and issuing loan depend on three variables: liquidity level, drift of the liquidity level and the interest rate. Given as initial conditions the liquidity level y and the interest rate r, we denote by $\mathcal{A}(y)$ the set of all admissible investment costs and the debt levels (I, D). The optimal investment-debt problem under continuous stochastic interest rate is to maximize the value function J given by

$$J(y, r; I, D) = \mathbb{E}^{y,r}\left[\limsup_{t \to \infty}\left(\int_0^{t \wedge \mathsf{T}} \Lambda_t \, dI_t - \int_0^{t \wedge \mathsf{T}} \Lambda_t \, dD_t\right)\right], \text{ where } \Lambda_t = \exp\left(-\int_0^t r_s ds\right). \tag{6}$$

Λ_t is a stochastic discounting factor as explained by Cuchiero [29] and it is stipulated in [30] that the normal property of the constant interest rate is inherited by the integral $\int_0^t r_s ds$. The corresponding optimal value function is then defined as

$$v(y,r) = \sup_{(I,D)\in\mathcal{A}(y)} J(y,r;I,D) \tag{7}$$

and the optimal investment-debt strategy (I_t^*, D_t^*) is such that

$$J(y,r;I_t^*,D_t^*) = v(y,r). \tag{8}$$

The formulation of the value function as appearing in equation (6) has been adopted from the studies by Sethi and Taksar [31] and, Løkka and Zervos [32]. However, the formulation in this study is different as we have considered the continuous stochastic interest rate rather than constant interest rate. Akyildirim et al. [33] provide a good reference for stochastic interest rate formulation though in discrete version.

3. The Value Function

In this section we provide the mathematical characterization of the optimal value function analytically. Principally, our interest is to maximize the expected discounted investment fund minus the expected discounted issued debt over all investment and debt issuance strategies while the liquid value is maintained positive . The discounting factor, in this particular case, is a stochastic interest rate.

Following the standard theory of singular control we come up with this form of of the Hamilton-Jacobi-Bellman (HJB) equation

$$\max\{\mathcal{L}_{y,r}v(y,r) - rv(y,r), -v_y(y,r)+1, v_y(y,r)-1\} = 0, \quad y>0,\ r>0 \tag{9}$$

with the boundary condition $v(0,r) = 0$ and initial condition $v(y,0) = y$, where the operator $\mathcal{L}$ for v is defined as

$$\mathcal{L}_{y,r} := [\mu + \phi(I_t) - \psi(D_t)]\frac{\partial}{\partial y} + \frac{1}{2}\sigma^2\frac{\partial^2}{\partial y^2} - \frac{\partial}{\partial r}. \tag{10}$$

As we take into consideration the value of money over time, we assert that it is desirable to postpone the issuance of new debt as long as feasible. Therefore we claim that it is optimal to issue debt only when the liquid level is below the possible investment value Y_I. The investment on the other hand is done only when the interest rate is below the fixed threshold rate r_θ. This is also to say that the company's management should not attempt for any action given that the interest rate is above r_θ or when the highest possible debt to be issued at time t, αY_t can not raise the liquid level above Y_I.

In our analysis, we are going to consider the non-negative solutions of equation (9) in C^2 as in [34]. In fact equation (9) is a dynamic programming equation characterized by

$$\mathcal{L}_{y,r} v(y,r) - rv(y,r) = 0, \quad \text{for} \quad 0 < y < Y_I - \alpha Y \text{ or } r \geq r_\theta, \tag{11}$$

$$v_y(y,r) = 1, \quad \text{for} \quad y \geq Y_I - \alpha Y \text{ and } 0 < r < r_\theta, \tag{12}$$

$$v(0,r) = 0. \tag{13}$$

Then it is well observed that equation (11) is a linear second order partial differential equation (PDE) of parabolic type with boundary conditions from equations (12) and (13). Equation (11) can also be presented in expanded and simplified form as

$$-v_r + \frac{1}{2}\sigma^2 v_{yy} + \hat{\mu} v_y - rv = 0, \tag{14}$$

where $\hat{\mu}$ stands for the drift expression $\mu + \phi(I_t) - \psi(D_t)$. Equations of this nature appear frequently in fluid and thermo-dynamics models as convection-diffusion models or reactive contaminant transport models; see e.g. [35] and [36]. However, apart from the difference in interpretation of the terms from equation (14), the third term is usually negative and the forth term has a constant coefficient for those models.

Since we are interested in showing how the liquidity level y, the drift $\hat{\mu}$ and the interest rate r influence the investment and debt decisions, we just go straight to solve the PDE in (14). The solution will show how the optimal value function v is influenced by the said variables. Prior to solve for v we show that there always exist a solution for (14) by transforming the equation into a simple diffusion equation (i.e the heat equation in thermodynamics) which has a solution. Later after solving for v we will show that the solution is unique.

Lemma 3.1. *Equation* (14) *can be systematically transformed into a simple diffusion equation which indicates the existence of solution.*

Proof. Transforming the function v into u through $u = ve^{\frac{1}{2}r^2}$ we have

$$v = e^{-\frac{1}{2}r^2}u, \quad v_r = -re^{-\frac{1}{2}r^2}u + e^{-\frac{1}{2}r^2}u_r, \quad v_y = e^{-\frac{1}{2}r^2}u_y, \quad v_{yy} = e^{-\frac{1}{2}r^2}u_{yy}.$$

Substituting for v in equation (14) we obtain

$$-u_r + \frac{1}{2}\beta^2 u_{yy} + \hat{\alpha}u_y = 0, \quad y > 0,\ r > 0. \tag{15}$$

Next we transform the independent variables y, r to $\eta = y - \hat{\alpha}r$, $\rho = r$ which leads to a simple diffusion equation under w,

$$-w_\rho + \frac{1}{2}\beta^2 w_{\eta\eta} = 0, \quad \eta > 0,\ \rho > 0. \tag{16}$$

This can be written as

$$w_r - \frac{1}{2}\beta^2 w_{yy} = 0, \quad y > 0,\ r > 0, \tag{17}$$

with a complete transformation given by

$$v(y,r) = w(y,r)e^{\frac{\hat{\alpha}}{\beta^2}(y-\hat{\alpha}r)-\frac{r^2}{2}}. \tag{18}$$

We depart from what is common in most of the literature whereby the optimal value function is just described by the known properties such as the concavity property. In this study we describe the optimal value function by presenting it explicitly in terms of the liquidity level y, the drift $\hat{\mu}$, the interest rate r and the volatility parameter σ. We apply Fourier transformation method in solving the PDE in (14).

Now equation (14) can be presented in a simple form as

$$\mathcal{L}_{y,r}v - rv = 0 \tag{19}$$

which can also be written as

$$v = \mathcal{L}_{y,r}^{-1} rv, \tag{20}$$

where the inverse differential operator $\mathcal{L}_{y,r}^{-1}$ is given by the following convolutional integral

$$\mathcal{L}_{y,r}^{-1} rv = \int_0^r F_{r-s} rv \, ds \tag{21}$$

in which F_r is the strongly continuous semigroup associated with (14) and as it is adopted from [35] has the form

$$Fg = \frac{1}{\sqrt{2\pi\sigma^2 r}} \int_{-\infty}^{\infty} e^{\frac{(y+\hat{\mu}r-t)^2}{2\sigma^2 r}} g \, dt. \tag{22}$$

This imply that

$$\mathcal{L}_{y,r}^{-1} rv = \int_0^r \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2(r-s)}} e^{\frac{(y+\hat{\mu}(r-s)-t)^2}{2\sigma^2(r-s)}} (r-s)v \, dt \, ds. \tag{23}$$

Next we attempt expansion of v in (23) above as the series $v = v_0 + v_1 + v_2 + \cdots$. Considering the original equation (11) and let v_i be the given initial value for the function v, the first term in our series is

$$v_0 = \mathcal{L}_{y,r}^{-1} v_i = \frac{v_i e^{\frac{(y+\hat{\mu}r)^2}{2\sigma^2 r}}}{\sqrt{2\pi\sigma^2 r}}. \tag{24}$$

The second term is

$$v_1 = \mathcal{L}_{y,r}^{-1} rv_0 = -\frac{r^2 v_i e^{\frac{(y+\hat{\mu}r)^2}{2\sigma^2 r}}}{\sqrt{2\pi\sigma^2 r}}. \tag{25}$$

The third term is

$$v_2 = \mathcal{L}_{y,r}^{-1} rv_1 = \frac{r^4 v_i e^{\frac{(y+\hat{\mu}r)^2}{2\sigma^2 r}}}{2\sqrt{2\pi\sigma^2 r}}. \tag{26}$$

Generally, the n$^{\text{th}}$ term has the following form

$$v_n = \mathcal{L}_{y,r}^{-1} rv_{n-1} = (-1)^n \frac{r^{2n}}{n!} \frac{v_i e^{\frac{(y+\hat{\mu}r)^2}{2\sigma^2 r}}}{2\sqrt{2\pi\sigma^2 r}}. \tag{27}$$

We sum the infinity terms and obtain the required solution

$$v(y,r) = \frac{v_i e^{\frac{(y+\hat{\mu}r)^2}{2\sigma^2 r} - r^2}}{\sqrt{2\pi\sigma^2 r}}. \tag{28}$$

This solution will be very important in the numerical experiments.

To prove uniqueness of the solution to our linear second order parabolic PDE, we rely on the Maximum Principle. We first state the following lemma which is actually a summary of the

Strong Maximum Principle as described and proved in [37].

Lemma 3.2. [37] *Consider the operator $\mathcal{L}$ such that*

$$\mathcal{L}u := \sum_{i,j=1}^{n} a_{ij}(t,x)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b(t,x)\frac{\partial u}{\partial x_i} + c(t,x)u - \frac{\partial u}{\partial t} \tag{29}$$

where the argument (t,x) vary in an $(n+1)$ dimensional set $S = (0,T] \times \Omega$ and $\Omega \subset \mathbb{R}^n$ is a bounded domain. Then $\mathcal{L}u = 0$ has unique solution if

(i) $\mathcal{L}$ is parabolic in S

(ii) The coefficients of $\mathcal{L}$ are continuous in S

(iii) $c(t,x) \leq 0$ in S

(iv) u has a positive maximum in S.

Theorem 3.1. *The partial differential equation as presented in* (11) *has unique solution.*

Proof. We define a new operator $\mathcal{L}$ on v from (11) as

$$\mathcal{L}v := \frac{1}{2}\sigma^2\frac{\partial^2 v}{\partial y^2} + \hat{\mu}\frac{\partial v}{\partial y} - rv - \frac{\partial v}{\partial r}. \tag{30}$$

We see that (r,y) vary in a two-dimensional set $\mathbb{R}^{2+}$. We show that the conditions (i)-(iv) given in Lemma 3.2 hold for $\mathcal{L}$.

(i) By inspecting the coefficients it is easily found that $\mathcal{L}$ is parabolic in $\mathbb{R}^{2+}$

(ii) The coefficients $\frac{1}{2}\sigma^2$ and -1 are constants, and $\hat{\mu}$ and $-r$ are variables which can assume values at every point (r,y). So the coefficients are continuous in $\mathbb{R}^{2+}$

(iii) $-r \leq 0$ in $\mathbb{R}^{2+}$ since $r \in \mathbb{R}^+$

(iv) The optimal value function v as found in equation (28) has always the maximum in $\mathbb{R}^{2+}$.

Thus the solution for the partial differential equation (11) is unique.

Corollary 3.2. *Consider the optimization problem of the value function $J(y,r;I,D)$ over all strategies (I,D) in the admissible set $\mathcal{A}(y)$. Then the unique solution v as obtained in equation* (28) *serves as the optimal value function to the HJB equation* (9).

4. Numerical experiments and discussion of results

In this section, we mainly provide a description on how the optimal value function v varies over the liquidity level y, the drift $\hat{\mu}$ and the interest rate r. The combination of y, $\hat{\mu}$ and r plays a major role in the investment and debt decisions of a firm as aforementioned. The numerics will therefore be based on the solution previously obtained in equation (28). Since assessing the variation of v over y, $\hat{\mu}$ and r at once would not appeal visually, we start by studying the variation of v in pairs of the variables. Next we present how v varies over each of the independent variables while others are given values of some of their extremities. This approach is useful for determination of some general or theoretical decision making criteria. The values of the parameters used in these numerical experiments have been inherited from the study by Pierre et al. [17] while the interest rates are the result of estimation.

The Figures 1, 2 and 3 are the 3D plots that describe the variation of v over the variables y, $\hat{\mu}$ and r in pairs. In Figure 1 where v is plotted against $\hat{\mu}$ and y, we observe that v increases exponentially in both $\hat{\mu}$ and y. In general, this suggests that the optimal time to take loan and/or invest is when the drift $\hat{\mu}$ is high and the liquidity level y is also high.

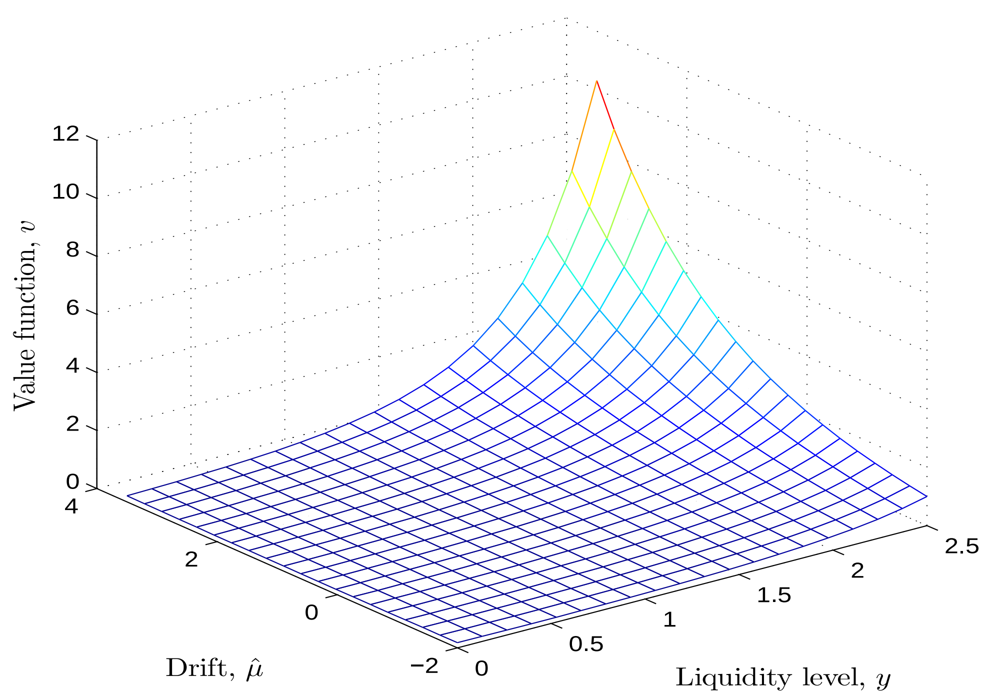

FIGURE 1. Dependence of the value function v on the drift $\hat{\mu}$ and liquidity level y with $r = 0.2$, $v_i = 0.5$ and $\sigma = 2.5$.

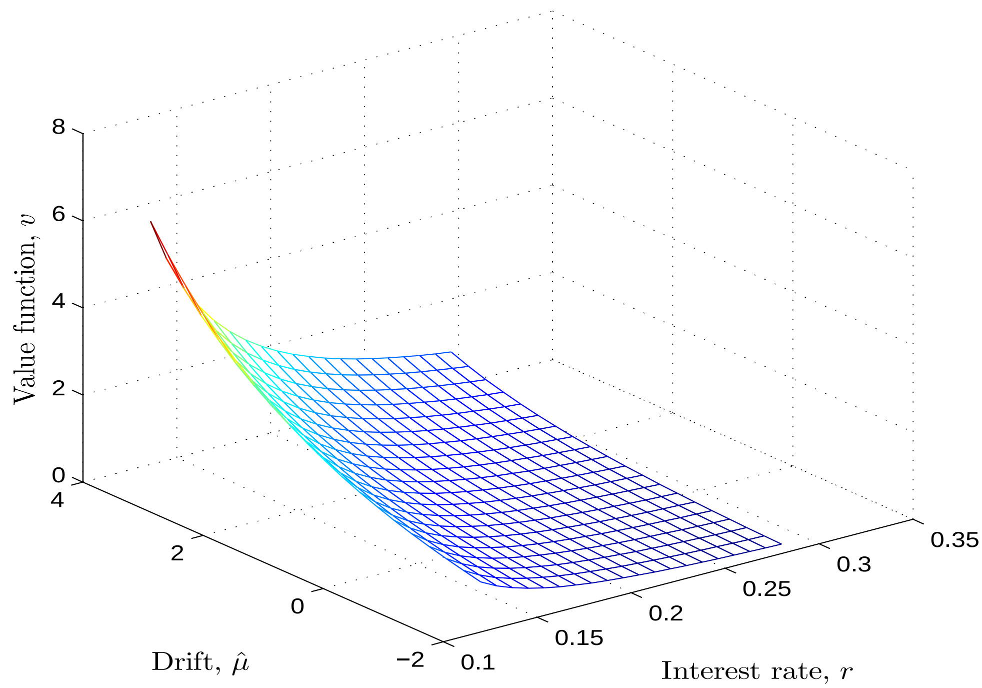

FIGURE 2. Dependence of the value function v on the drift $\hat{\mu}$ and interest rate r with $y = 1.8$, $v_i = 0.5$ and $\sigma = 2.5$.

In Figure 2 the value function v is surfaced over the drift $\hat{\mu}$ and the interest rate r. It is reveled that v increases as $\hat{\mu}$ increases but goes down as r increases. Generally, this means that having high drift in liquidity value of a company combined with low interest rate in the market is good condition for taking loan and for investing. A similar observation is found in Figure 3 in which v is plotted over y and r. We also learn that the good time to take loan and/or invest is when the liquid level is high and the interest rate is low.

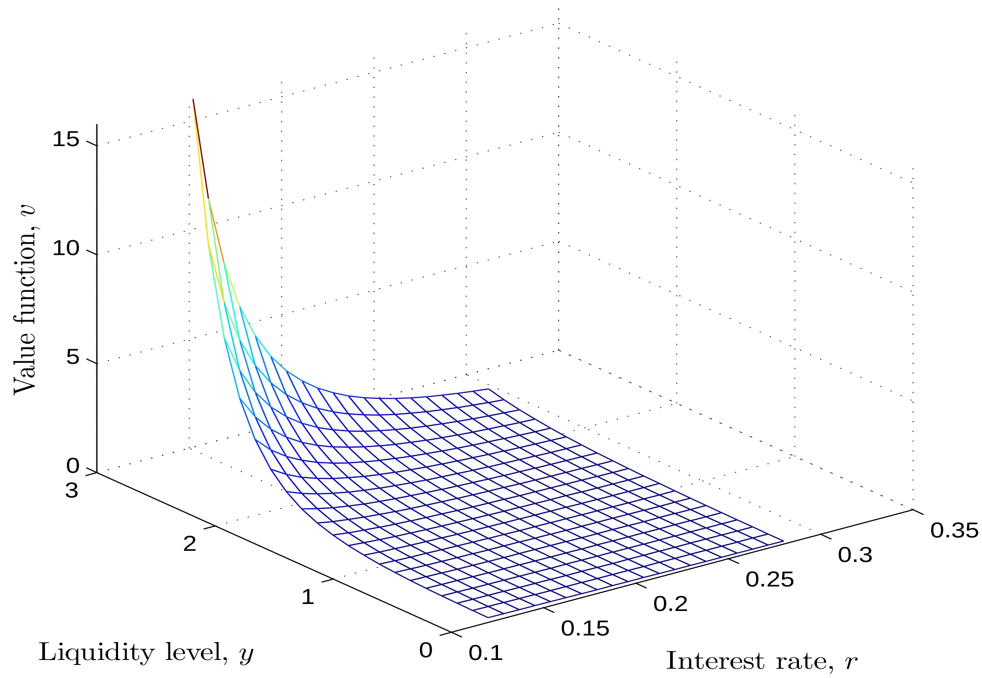

FIGURE 3. Dependence of the value function v on the interest rate r and liquidity level y with $\hat{\mu} = 0.5$, $v_i = 0.5$ and $\sigma = 2.5$.

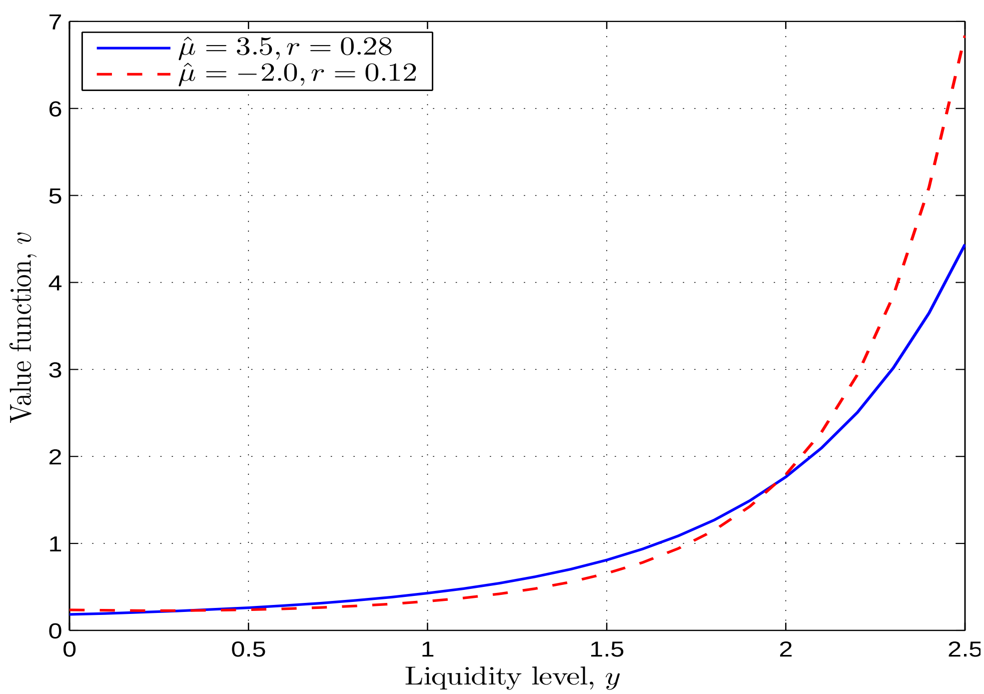

FIGURE 4. Variation of the value function v over liquidity level y for different values of the drift $\hat{\mu}$ and interest rate r with $v_i = 0.5$ and $\sigma = 2.5$.

In Figure 4 we have curves that represent how the value function v varies over the liquidity level for the extremities of the drift and the interest rates. The extremities are paired with opposing influence on the value function. We obtain two important points, $y_1 \approx 0.5$ and $y_2 \approx 2.0$, where the two curves cross each other. This imply that it is not optimal to take loan and invest when the liquidity level of y is below y_1 as the amount of cash will not be able to suffice investment. Also it is not optimal to take loan and invest when y is above y_2 as the amount of cash at hand is already sufficient to further investment without loan. Actually, the point y_1 is Y_L and the point y_2 is Y_I as described in section 2.

Figure 5 is about the value function over the drift $\hat{\mu}$ for chosen extremities for r and y in pairs with opposing influence on v. The two curves cross each other at the point where $\hat{\mu}$ has approximately the value of -0.7. This is a point of decision about $\hat{\mu}$, whether or not to take loan and/or invest. Having $\hat{\mu} = \mu + \phi(I_t) - \psi(D_t)$ below this value is not good moment for taking loan and/or invest.

We also apply similar approaches in Figure 6 and find that the decision making point is $r_\theta \approx 0.17$. So it is optimal to take loan and/or invest when the interest rate is below 17%.

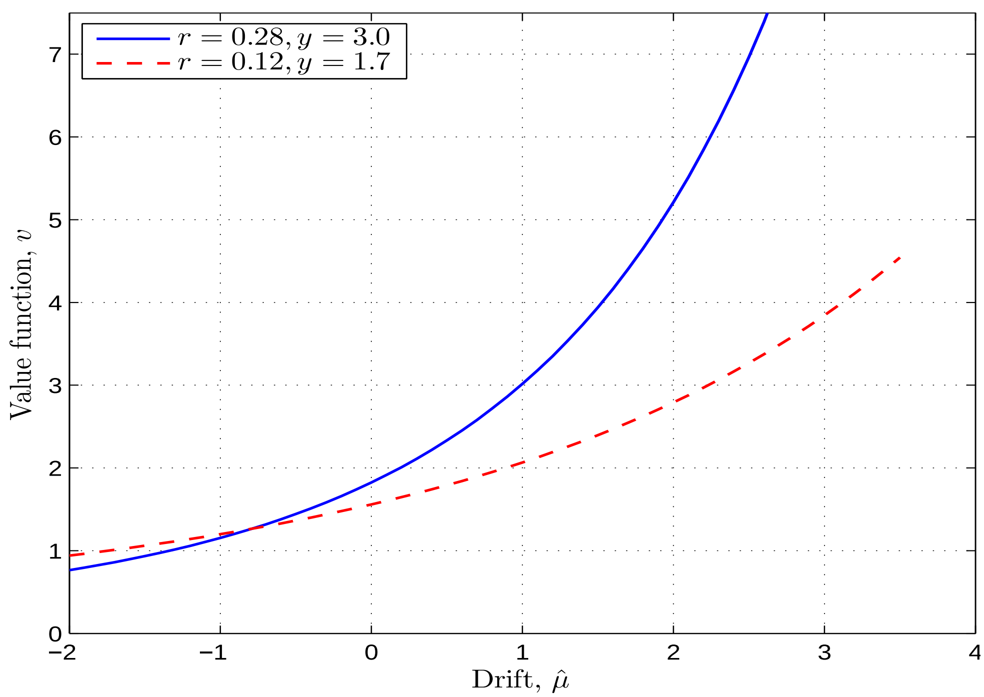

FIGURE 5. Variation of the value function v over the drift $\hat{\mu}$ for different values of interest rate r and liquidity level y with $v_i = 0.5$ and $\sigma = 2.5$.

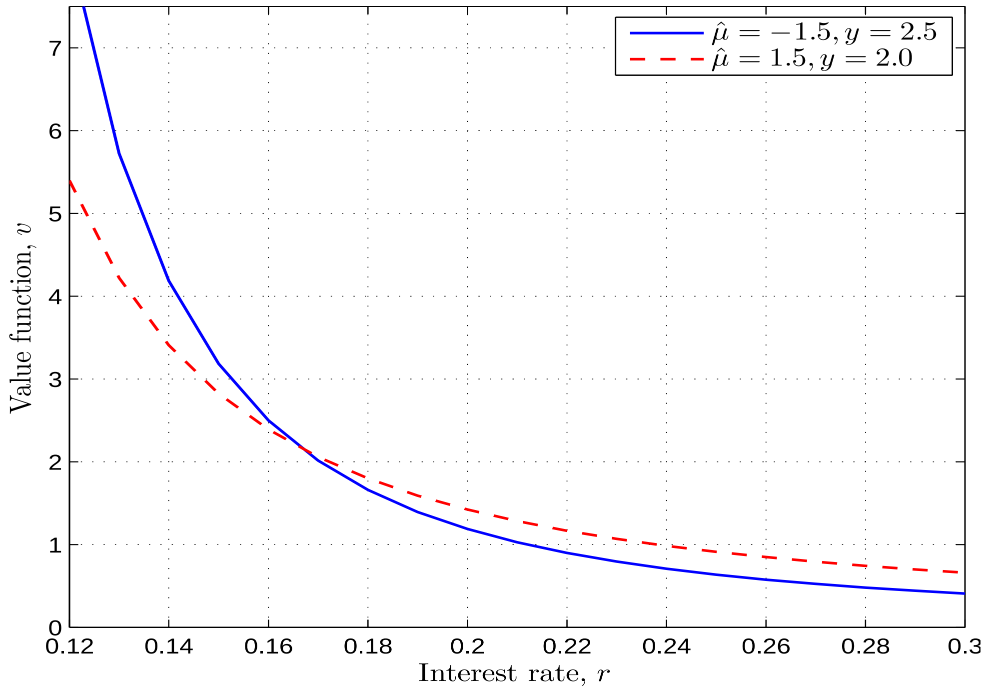

FIGURE 6. Variation of the value function v over the interest rate r for different values of the drift $\hat{\mu}$ and liquidity level y with $v_i = 0.5$ and $\sigma = 2.5$.

5. Conclusion, Recommendation and Possible extensions

We have formulated an approach that can be applied by company managers in optimizing the investment and debt policies with the consideration of randomly fluctuating interest rates. The approach is more suitable for firms in developing countries where the economy is normally unstable and for firms with liquidity levels that can be modeled by SDE. Firm managers need be confident with the endogenous and exogenous business status as they make investment and debt decisions.

Generally, we have been able to show the variation of the optimal value function over the liquidity level, liquidity drift and the interest rates. Specifically, we showed that the value function exponentially increases as the liquidity level increases and as the liquidity drift increases. However, the value function decreases as the interest rates become higher and higher. This means that the best time to take loan and/or invest is when the liquidity level and the liquidity drift of a company are sufficiently high while the market interest rates are reasonably low. Moreover, we found two limiting points over the liquidity level domain where below the lower point and above the upper point the decision to take loan and invest is not optimal. This is because below the lower point the amount of cash will not be able to suffice investment and above the upper point the amount of cash at hand is already sufficient to further investment without loan. For the case of the drift we found a point above which taking loan and invest is optimal while having an interest rate below the threshold interest rate suggests good condition for taking loan and/or invest. Therefore, as the financial policy makers have to ensure that the interest rates are as low as possible, firm managers on the other hand have the role to consider endogenous characteristics of their firms such as liquidity level and liquidity drift in making investment and debt decisions.

The upshot of this study open doors for further investigation on the optimal decision on the firm financial management. For example, inclusion of the dividend variable in the model formulation will possibly improve the results and add more value to the findings. The extended model can also consider dividend smoothing under stochastic interest rate settings. We reserve all these for future consideration.

Conflict of Interests

The authors declare that there is no conflict of interests.

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