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## VALUATION AND MANAGEMENT OF FLOW OF INTERNALLY GENERATED REVENUE AND FEDERAL GOVERNMENT MONETARY ALLOCATIONS

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**Abstract.** This paper examines the valuation and management of inflows of internally generated revenue and monetary allocations from the federal government (FG) to a state government (SG). The SG decides to invest part of the internally generated revenue and monetary allocations into a cash account and a stock in order to make more money for the state. The aim of the study is to determine the value of total wealth (i.e., the value of wealth from the investment and cash inflows) that will accrued to the SG over a period of time. The present and terminal values of the SG total revenue was obtained. It was found that investment of the internally generated revenue into stocks and cash account should be encourage to enhance maximum revenue for infrastructural development of the state.

**Keywords:** Internally generated revenue, Federal government, Monetary allocations, State government.

**2000 AMS Subject Classification:** 91B28, 91B30, 91B70, 93E20

### 1. Introduction

In this study, we consider the valuation and management of inflows of state government internally generated revenue (SGIGR) and allocation from Federation Account (FA) of a state. Many SG finances have been dwindling due to decreasing trends and values of

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the pool of distributions in the FA in Nigeria. These have considerably reduced the state government's share of the FA. As a result, the FG allocation to states can no longer take care of the SG expenditures. Having realized that the monetary allocation from the FA are no longer adequate to finance their expenditures, many SGs have decided to tighten the economic belts of her state by focusing on other sources to raise fund, among which are investment proceeds. Many SG hold shares in various companies and establishment in and outside the country. The state ministry of finance on behalf of the SG purchased these shares. The dividends that accrued from the shares form an integral part of the SGIGR. In this paper, we assume that the SG choose to invest part of the cash inflows into stock and cash account overtime. Since the cash inflows are amount to be received sometime in the future, we choose to discount them.

[3] designed an optimal investment strategy in a portfolio of assets subject to a multiplicative Brownian motion. They stipulates investment strategy provides the maximal typical long-term growth rate of investor's capital. They obtained optimal fraction of capital that an investor should keep in risky assets and weights of different assets in an optimal portfolio. They found that both average return and volatility of an asset are relevant indicators for determining its optimal weight. [2] stipulated that government policy is typically targeted heavily on investment. [1] considered optimal investment and consumption decision of a certain constant relative risk averse (CRRA) investor who faces proportional transaction costs in a finite time horizon. They found that the problem gave rise to two free boundaries for the optimal buying and selling strategies. [5] studied optimal investment in a financial market that is characterized by a finite number of assets from a signal processing perspective. They investigated how an investor should distribute capital over the various assets and determined when to reallocate the distribution of the funds over the assets to maximize the cumulative wealth over any investment period. They developed portfolio selection algorithm that maximizes the expected cumulative wealth in independent and identically distributed two-asset discrete-time markets where the market levies proportional transaction costs in buying and selling stocks. [4] considered the optimal portfolio management of the accumulation phase of a DC pension

scheme. Furthermore, the variational form of portfolio strategies were obtained. In this paper, we consider the valuation of the SGIGR and the allocation from FA. We adopt the dynamic programming approach in the optimization of the expected wealth the will accrued to the SG up to the terminal time.

The remainder of this paper is organized as follows. In section 2, we present the financial market models and the wealth dynamic of the SG at time  $t$ . In section 3, we present the valuation of the SG wealth in the investment strategy. In section 4, we present the optimization of the SG portfolio, expected wealth that will accrued to SG at time  $t$  as well as the terminal wealth of the SG. Section 5 presents the variance of the value of terminal wealth of the SG. Finally, section 6 conclude the paper.

## 2. The Models

Let  $W(t)$ ,  $0 \leq t \leq T$  be a 1-dimensional Brownian motion, defined on a probability space  $(\Omega, \mathbb{F}, \mathbb{P})$ , where  $\mathbb{P}$  is the real world probability measure and  $\sigma$  is the volatility of stock with respect to change in  $W(t)$ .  $\mu > 0$  is the appreciation rate for stock.

### 2.1. Financial Models

In this paper, we assume that the some proportion of the flow of funds are invested in a market that is characterized by a risk-free asset (cash account) and a risky asset. Therefore, the dynamics of the underlying assets are given in (1) and (2):

$$(1) \quad \begin{cases} dB(t) = rB(t)dt, \\ B(0) = 1, \end{cases}$$

$$(2) \quad \begin{cases} dS(t) = S(t) (\mu dt + \sigma dW(t)), \\ S(0) = s_0 > 0, \end{cases}$$

where,

$r$  is the nominal interest rate,

$B(t)$  is the price process of the cash account at time  $t$ ,

$S(t)$  is stock price process at time  $t$ .

**Definition 1.** *The internally generated revenue for the state government is define by*

$$(3) \quad I(t) = I_0 \exp(\beta t),$$

where,  $I_0$  is the initial internally generated revenue by the state government and  $\beta$  is the expected growth rate of the internally generated revenue which depends on the growth of the economics of the people of the state.

**Definition 2.** *The allocation of resource from the federal government to the state is define by*

$$(4) \quad A(t) = a \exp(\alpha t),$$

where,  $a$  is the initial allocation of resource from the FG to the SG and  $\alpha$  is the expected growth rate of allocation of resource from the FG to the SG. It depends on the growth of the economics of the nation.

### 3. The Wealth Dynamics

Let  $\lambda$  be the proportion of the state internally generated revenue and allocation from federal government that should be invested into cash account and stock, then the amount to be invested is  $\lambda(I(t) + A(t))$  at time  $t$ . Let  $X(t)$  be the wealth process of the SG at time  $t$ . Let  $\Delta(t)$  be the portfolio process in stock at time  $t$ , then  $\Delta_0(t) = 1 - \Delta(t)$  is the proportion of wealth invested in cash account at time  $t$ . Therefore, the wealth process is governs by the stochastic differential equation (SDE):

$$(5) \quad \begin{cases} dX(t) = (X(t)(r + \Delta(t)(\mu - r)) + \lambda(I(t) + A(t))) dt + X(t)\Delta(t)\sigma dW(t), \\ X(0) = x_0. \end{cases}$$

**Definition 3.** *The value of the flows of the internally generated revenue and allocations at time  $t$  is define as*

$$(6) \quad \Psi(t) = \int_0^T \frac{\Lambda(s)}{\Lambda(t)} (I(s) + A(s)) ds,$$

where,  $\Lambda(t) = \exp(-rt)$ .

**Proposition 1.** *Let  $\Psi(t)$  be the value of flow of the internally generated revenue and allocations at time  $t$ , then*

$$(7) \quad \Psi(t) = \frac{I_0 \exp(rt)}{\beta - r} (\exp((\beta - r)T) - 1) + \frac{a \exp(rt)}{\alpha - r} (\exp((\alpha - r)T) - 1).$$

*Proof.* By definition 3, we have

$$\begin{aligned} \Psi(t) &= \int_0^T \frac{\Lambda(s)}{\Lambda(t)} (I(s) + A(s)) ds \\ &= \int_0^T \exp(-r(s - t)) (I_0 \exp(\beta s) + a \exp(\alpha s)) ds \\ &= I_0 \exp(rt) \int_0^T \exp(\beta - r)s ds + a \exp(rt) \int_0^T \exp((\alpha - r)s) ds \end{aligned}$$

Integrating, we have

$$(8) \quad \Psi(t) = \frac{I_0 \exp(rt)}{\beta - r} (\exp((\beta - r)T) - 1) + \frac{a \exp(rt)}{\alpha - r} (\exp((\alpha - r)T) - 1).$$

□

### 3. The Valuation of the SG Wealth in the Investment

In this section, we consider the valuation of the SG wealth that is invested in the financial market at time  $t$ .

**Definition 4.** *The value of wealth of the SG is define by*

$$(9) \quad V(t) = X(t) + \lambda \Psi(t).$$

Finding the differential of both sides of (9), we have

$$(10) \quad \begin{cases} dV(t) = dX(t) + \lambda z r \exp(rt) dt, \\ V(0) = x_0 + \lambda \Psi(0) = v_0, \end{cases}$$

where,  $z = \frac{I_0(\exp((\beta - r)T) - 1)}{\beta - r} + \frac{a(\exp((\alpha - r)T) - 1)}{\alpha - r}$ .

Substituting, (5) into (10), we have

$$(11) \quad \begin{cases} dV(t) = (X(t)(r + \Delta(t)(\mu - r)) + \lambda(I(t) + A(t)) + \lambda zr \exp(rt)) dt + X(t)\Delta(t)\sigma dW(t), \\ V(0) = v_0. \end{cases}$$

Using (3) and (4) in (11), we have

$$(12) \quad \begin{cases} dV(t) = (X(t)(r + \Delta(t)(\mu - r)) + \lambda(I_0 \exp(\beta t) + a \exp(\alpha t) + zr \exp(rt))) dt + X(t)\Delta(t)\sigma dW(t), \\ V(0) = v_0. \end{cases}$$

Setting  $\Phi(t) = I_0 \exp(\beta t) + a \exp(\alpha t) + zr \exp(rt)$  in (12), we have

$$(13) \quad \begin{cases} dV(t) = (X(t)(r + \Delta(t)(\mu - r)) + \lambda\Phi(t)) dt + X(t)\Delta(t)\sigma dW(t), \\ V(0) = v_0. \end{cases}$$

#### 4. Optimization of the Portfolio process

In this section, we consider the optimal portfolio process for the SG. We define the general value function

$$J(t, v) = E[u(V(T)) | X(t) = x]$$

where  $u(V(t))$  is the path of  $V(t)$  given the portfolio strategy  $\Delta(t)$ . Define  $A(V)$  to be the set of all admissible portfolio strategy that are  $\mathbb{F}_V$ -progressively measurable, and let  $U(V(t))$  be a concave function in  $V(t)$  such that

$$(14) \quad U(V(t)) = \sup_{\Delta \in A(V)} E[U(V(T)) | X(t) = x] = \sup_{\Delta \in A(V)} J(t, v).$$

Then  $U(V(t))$  satisfies the HJB equation

$$(15) \quad \begin{cases} U_t + \max_{\Delta} H(t) = 0 \\ \text{subject to: } U(v) = \frac{v^\gamma}{\gamma}. \end{cases}$$

Since  $U$  is a concave function in  $V(t)$  and  $U(V(t)) \in C^{1,2}(\mathbb{R} \times [0, T])$ , then (15) is well defined, where,

$$(16) \quad H(t) = rxU_x + \Delta(t)(\mu - r)xU_x + \lambda\Phi(t)U_x + \frac{1}{2}x^2\Delta(t)^2\sigma^2U_{xx}.$$

From (16), we have

$$(17) \quad \Delta^*(t) = \frac{-(\mu - r)U_x}{x\sigma^2U_{xx}}.$$

But,  $U_x = x^{\gamma-1}$  and  $U_{xx} = (\gamma - 1)x^{\gamma-2}$ .

Therefore,

$$(18) \quad \Delta^*(t) = \frac{\mu - r}{(1 - \gamma)\sigma^2}.$$

Substituting (18) into (13), we have

$$(19) \quad \begin{cases} dV^*(t) = (X^*(t)(r + \frac{\theta^2}{1 - \gamma})) + \lambda\Phi(t)dt + X^*(t)\frac{\theta}{1 - \gamma}dW(t), \\ V(0) = v_0. \end{cases}$$

Taking the mathematical expectation of (19), we have

$$(20) \quad \begin{cases} dE(V^*(t)) = E[(\omega V^*(t) - g \exp(rt) + \lambda I_0 \exp(\beta t) + a\lambda \exp(\alpha t))]dt \\ E(V^*(0)) = v_0 \end{cases}$$

where,  $\omega = r + \frac{\theta^2}{1 - \gamma}$ ,  $g = \frac{\theta^2 \lambda z}{1 - \gamma}$ .

Therefore,

$$(21) \quad \begin{aligned} E(V^*(t)) &= v_0 \exp(\omega t) + \frac{g(\exp(\omega t) - \exp(rt))}{r - \omega} \\ &+ \frac{\lambda I_0(\exp(\omega t) - \exp(\beta t))}{\omega - \beta} + \frac{a\lambda(\exp(\omega t) - \exp(\alpha t))}{\omega - \alpha}. \end{aligned}$$

At  $t = T$ , we have

$$(22) \quad \begin{aligned} E(V^*(T)) &= v_0 \exp(\omega T) + \frac{g(\exp(\omega T) - \exp(rT))}{r - \omega} + \\ &\frac{\lambda I_0(\exp(\omega T) - \exp(\beta T))}{\omega - \beta} + \frac{a\lambda(\exp(\omega T) - \exp(\alpha T))}{\omega - \alpha}. \end{aligned}$$

Therefore, the expected optimal value of terminal wealth for the SG is given in (22).

Therefore, the optimal value of terminal wealth for the SG is the sum of the fund,  $v_0 \exp(\omega T)$  that the IC would get investing the whole portfolio always in the risk-less asset

and the risky assets, plus the term,  $\frac{g(\exp(\omega T) - \exp(rT))}{r - \omega}$  that depend both on the goodness of the risky assets with respect to the risk-less asset, plus the term,  $\frac{\lambda I_0(\exp(\omega T) - \exp(\beta T))}{\omega - \beta}$  that depend on the risk-less asset, the risky assets and the internally generated revenue by the SG, plus the term  $\frac{a\lambda(\exp(\omega T) - \exp(\alpha T))}{\omega - \alpha}$  that depend on the risk-less asset, the risky assets and the FG allocation to the SG. Thus, we observe by the definition of  $\theta$  (i.e., *Sharpe ratio*), that the higher the *Sharpe ratio* of the risky asset,  $\theta$  the higher the expected optimal value of terminal wealth, everything else being equal.

**Definition 5.** *The total value of expected wealth that will accrued to the SG at time  $t$  is define as*

$$(23) \quad Y(t) = E(V(t)) + (1 - \lambda)\Psi(t),$$

where,  $\Psi(t)$  satisfies (6). Hence, the total optimal value of expected wealth that will accrue to the SG is given by

$$(24) \quad Y(t) = v_0 \exp(\omega t) + \frac{g(\exp(\omega t) - \exp(rt))}{r - \omega} + \frac{\lambda I_0(\exp(\omega t) - \exp(\beta t))}{\omega - \beta} + \frac{a\lambda(\exp(\omega t) - \exp(\alpha t))}{\omega - \alpha} + (1 - \lambda)z \exp(rt).$$

At  $t = 0$ , we have

$$(25) \quad Y(0) = v_0 + (1 - \lambda)z.$$

(25) is the present value of the SG total revenue. At  $t = T$ , we have

$$(26) \quad Y(T) = v_0 \exp(\omega T) + \frac{g(\exp(\omega T) - \exp(rT))}{r - \omega} + \frac{\lambda I_0(\exp(\omega T) - \exp(\beta T))}{\omega - \beta} + \frac{a\lambda(\exp(\omega T) - \exp(\alpha T))}{\omega - \alpha} + (1 - \lambda)z \exp(rT).$$

(26) represents the terminal value of the SG total revenue.

If the SG is insensitive to investment, we have that the value of wealth that will accrue to it at time  $t$  to be

$$(27) \quad Y(t) = x_0 \exp(\omega t) + z \exp(rt).$$



(27) was obtained by setting  $\lambda = 0$  in (24).  $x_0$  is the initial amount that predecessor left in the SG portfolio. If  $x_0 = 0$ , it implies that the predecessor spent all amount in the SG portfolio. In that case, (27) reduces to

$$(28) \quad Y(t) = z \exp(rt).$$

If we set  $\lambda = 1$ , (24) becomes

$$(29) \quad Y(t) = v_0 \exp(\omega t) + \frac{g(\exp(\omega t) - \exp(rt))}{r - \omega} + \frac{I_0(\exp(\omega t) - \exp(\beta t))}{\omega - \beta} + \frac{a(\exp(\omega t) - \exp(\alpha t))}{\omega - \alpha}.$$

Observe that when  $\lambda = 1$ ,  $Y(t) = V(t)$ . This means that the flow of internally generated revenue and the FG allocation to SG are all invested in the financial market at time  $t$ .

## 5. The Variance of the Value of Wealth of the SG

In this section, we consider the variance of the value of wealth of the SG. Applying *Itô* lemma on (19), we have

$$(30) \quad dV^{*2}(t) = (2\omega V^{*2}(t) + 2V^*(t)(\lambda I_0 \exp(\beta t) - g \exp(rt) + a \lambda \exp(\alpha t) + X^2(t)h^2) dt + 2V^*(t)X^*(t)hdW(t),$$

where  $h = \frac{\theta}{1 - \gamma}$ .

$$(31) \quad \begin{cases} dE(V^{*2}(t)) = E(2\omega(V^{*2}(t)) + 2(V^*(t))(\lambda I_0 \exp(\beta t) - g \exp(rt) + a \lambda \exp(\alpha t) + X^2(t)h^2) dt, \\ E(V^{*2}(0)) = v_0^2. \end{cases}$$

**Lemma 1.** *Given that  $V(t)$  satisfies (9), then  $X^2(t) = V^2(t) - 2\lambda\Psi(t)V(t) + \lambda^2\Psi^2(t)$ .*

*Proof.* By definition,  $V(t) = X(t) + \lambda\Psi(t)$ . Then,

$$\begin{aligned} V^2(t) &= (X(t) + \lambda\Psi(t))^2 \\ &= X^2(t) + 2\lambda X(t)\Psi(t) + \lambda^2\Psi^2(t) \end{aligned}$$

Therefore,

$$\begin{aligned}
X^2(t) &= V^2(t) - 2\lambda X(t)\Psi(t) - \lambda^2\Psi^2(t) \\
&= V^2(t) - \lambda\Psi(t)(X(t) + \lambda\Psi(t)) - \lambda X(t)\Psi(t) \\
&= V^2(t) - \lambda V(t)\Psi(t) - \lambda X(t)\Psi(t) - \lambda^2\Psi^2(t) + \lambda^2\Psi^2(t) \\
&= V^2(t) - \lambda V(t)\Psi(t) - \lambda\Psi(t)(X(t) + \lambda\Psi(t)) + \lambda^2\Psi^2(t) \\
&= V^2(t) - \lambda V(t)\Psi(t) - \lambda\Psi(t)V(t) + \lambda^2\Psi^2(t) \\
&= V^2(t) - 2\lambda V(t)\Psi(t) + \lambda^2\Psi^2(t).
\end{aligned}$$

□

By lemma (1), (31) becomes

$$(32) \quad \begin{cases} dE(V^{*2}(t)) = ((2\omega + h^2)E(V^{*2}(t)) + 2E(V^*(t))(\lambda I_0 \exp(\beta t) \\ - (g + z\lambda) \exp(rt) + a\lambda \exp(\alpha t)) + z^2\lambda^2 \exp(2rt))dt \\ E(V^{*2}(0)) = v_0^2 \end{cases}$$

Integrating (32), we have

$$(33) \quad \begin{aligned} E(V^{*2}(t)) &= \left( v_0^2 + \frac{z^2\lambda^2}{2r}(\exp(2rt) - 1) \right) \exp((2\omega + h^2)t) \\ &+ 2 \exp((2\omega + h^2)t) \int_0^t E(V^*(u))(\lambda I_0 \exp(\beta u) - (g + z\lambda) \exp(ru) + a\lambda \exp(\alpha u)) \exp(-(2\omega + h^2)u) du. \end{aligned}$$

At  $t = T$ , we have

$$(34) \quad \begin{aligned} E(V^{*2}(T)) &= \left( v_0^2 + \frac{z^2\lambda^2}{2r}(\exp(2rT) - 1) \right) \exp((2\omega + h^2)T) \\ &+ 2 \exp((2\omega + h^2)T) \int_0^T E(V^*(u))(\lambda I_0 \exp(\beta u) - (g + z\lambda) \exp(ru) + a\lambda \exp(\alpha u)) \exp(-(2\omega + h^2)u) du. \end{aligned}$$

Therefore, the variance of the expected terminal value of wealth of the SG is obtained

as

$$(35) \quad \begin{aligned} Var(V^*(T)) &= \left( v_0^2 + \frac{z^2\lambda^2}{2r}(\exp(2rT) - 1) \right) \exp((2\omega + h^2)T) \\ &+ 2 \exp((2\omega + h^2)T) \int_0^T E(V^*(u))(\lambda I_0 \exp(\beta u) - (g + z\lambda) \exp(ru) \\ &+ a\lambda \exp(\alpha u)) \exp(-(2\omega + h^2)u) du - (E(V^*(T)))^2. \end{aligned}$$

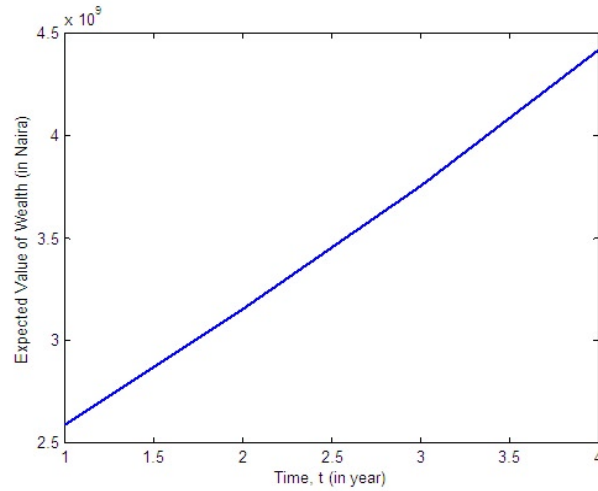


FIGURE 1. Expected Value of Wealth Accrued to SG for  $\lambda = 1$ . This figure was obtained by setting  $I_0 = 25,600,000$ ,  $a=452,370,000$ ,  $r = 0.01$ ,  $\beta = 0.02$ ,  $\alpha = 0.05$ ,  $\mu = 0.05$ ,  $\sigma = 0.35$ ,  $\gamma = 0.5$  and  $x_0 = 1$ .

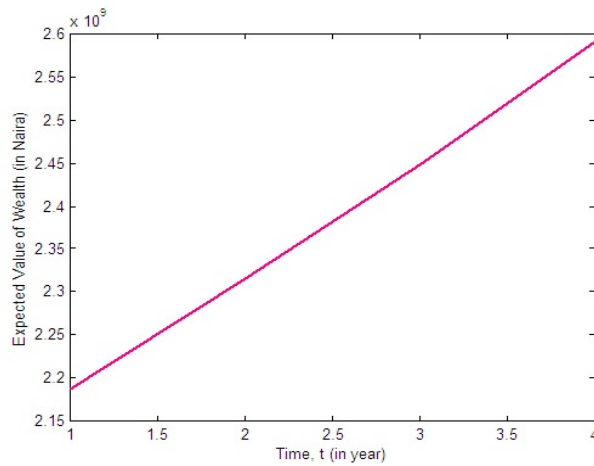


FIGURE 2. Expected Value of Wealth Accrued to SG for  $\lambda = 0.1$ . This figure was obtained by setting  $I_0 = 25,600,000$ ,  $a=452,370,000$ ,  $r = 0.01$ ,  $\beta = 0.02$ ,  $\alpha = 0.05$ ,  $\mu = 0.05$ ,  $\sigma = 0.35$ ,  $\gamma = 0.5$  and  $x_0 = 1$ .

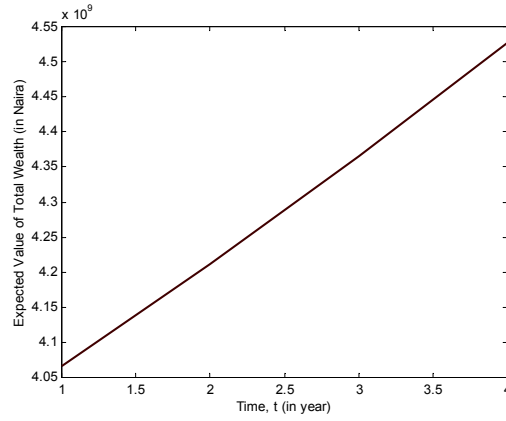


FIGURE 3. Expected Value of Total Wealth Accrued to SG for  $\lambda = 0.1$ . This figure was obtained by setting  $I_0 = 25,600,000$ ,  $a=452,370,000$ ,  $r = 0.01$ ,  $\beta = 0.02$ ,  $\alpha = 0.05$ ,  $\mu = 0.05$ ,  $\sigma = 0.35$ ,  $\gamma = 0.5$  and  $x_0 = 1$ .

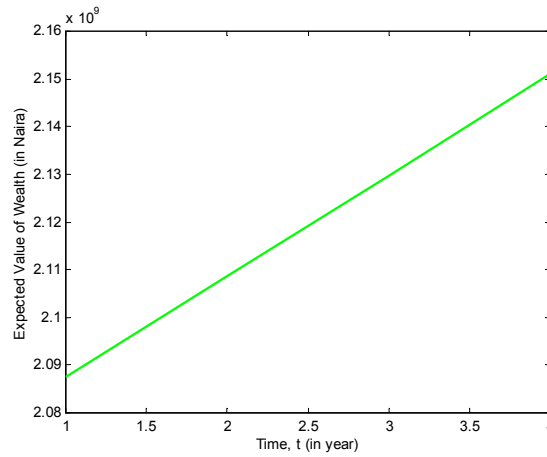


FIGURE 4. Expected Value of Wealth Accrued to SG where there is no investment. This figure was obtained by setting  $I_0 = 25,600,000$ ,  $a=452,370,000$ ,  $r = 0.01$ ,  $\beta = 0.02$ ,  $\alpha = 0.05$ .

**Table 1: Expected Value of SG Wealth in the Investment Plan**

$t$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$
	$\times 10^9 N$	$\times 10^9 N$	$\times 10^9 N$	$\times 10^9 N$	$\times 10^9 N$	$\times 10^9 N$
1	4.6809	4.7187	4.7566	4.7945	4.8324	4.8703
2	4.8933	4.9728	5.0523	5.1319	5.2114	5.2909
3	5.1160	5.2412	5.3663	5.4915	5.6167	5.7419
4	5.3495	5.5246	5.6897	5.8748	6.0499	6.2250
5	5.5943	5.8240	6.0536	6.2832	6.5128	6.7425
6	5.8511	6.1401	6.4292	6.7182	7.0072	7.2963
7	6.1203	6.4740	6.8277	7.1813	7.5350	7.8887
8	6.4026	6.8265	7.2504	7.6743	8.0982	8.5221

where,  $N$  denotes Naira and  $t$  time in years.

**Table 2: Expected Value of Total Wealth Accrued to SG**

$t$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$
	$\times 10^{10}N$	$\times 10^{10}N$	$\times 10^{10}N$	$\times 10^{10}N$	$\times 10^{10}N$	$\times 10^{10}N$
1	0.8752	0.8337	0.7923	0.7508	0.7094	0.6680
2	0.9005	0.8628	0.8250	0.7873	0.7496	0.7118
3	0.9269	0.8933	0.8597	0.8260	0.7924	0.7588
4	0.9544	0.9253	0.8962	0.8671	0.8380	0.8089
5	0.9831	0.9590	0.9349	0.9108	0.8867	0.8626
6	1.0131	0.9944	0.9758	0.9571	0.9385	0.9198
7	1.0443	1.0316	1.0190	1.0063	0.9937	0.9810
8	1.0767	1.0708	1.0646	1.0585	1.0524	1.0463

The value in Table 1 and 2 are obtained by setting  $I_0 = 25,600,000$ ,  $a = 452,370,000$ ,  $r = 0.01$ ,  $\beta = 0.02$ ,  $\alpha = 0.05$ ,  $\mu = 0.05$ ,  $\sigma = 0.35$ ,  $\gamma = 0.5$ ,  $x_0 = 1$  and  $\lambda$  is allow to vary.

**Table 3: Expected Value of Wealth of SG in IP at Different Values of  $\gamma$** 

$t$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$	$\gamma = 0.5$	$\gamma = 0.6$	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$
	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$
1	2.1646	2.1681	2.1726	2.1786	2.1871	2.1999	2.2214	2.2652	2.4022
2	2.2672	2.2745	2.2839	2.2965	2.3143	2.3413	2.3870	2.4815	2.7906
3	2.3750	2.3864	2.4010	2.4208	2.4487	2.4913	2.5642	2.7172	3.2402
4	2.4881	2.5039	2.5242	2.5517	2.5908	2.6506	2.7538	2.9740	3.7601

**Table 4: Expected Value of Total Wealth Accrued to SG at Different Values of  $\gamma$** 

$t$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$	$\gamma = 0.5$	$\gamma = 0.6$	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$
	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$	$\times 10^9N$
1	4.0433	4.0468	4.0514	4.0574	4.0659	4.0787	4.1002	4.1439	4.2810
2	4.1649	4.1722	4.1816	4.1942	4.2120	4.2389	4.2847	4.3791	4.6883
3	4.2917	4.3031	4.3179	4.3375	4.3655	4.4081	4.4809	4.6339	5.1569
4	4.4241	4.4398	4.4602	4.4877	4.5268	4.5866	4.6898	4.9100	5.6961

The value in Table 3 and 4 are obtained by setting  $I_0 = 25,600,000$ ,  $a = 452,370,000$ ,  $r = 0.01$ ,  $\beta = 0.02$ ,  $\alpha = 0.05$ ,  $\mu = 0.05$ ,  $\sigma = 0.35$ ,  $x_0 = 1$ ,  $\lambda = 0.1$  and  $\gamma$  is allow to vary.

Table 1 shows the expected wealth of the SG in the investment profile for 8 years. We found that as the value of  $\lambda$  increases the expected value of the SG wealth in the investment plan increases and the total wealth of the investor increases alongside, as it is shown in table 2. Table 2 shows the expected total wealth that will accrued to SG for the period of 8 years. Table 3 shows the expected value of wealth of SG in the investment profile at different values of  $\gamma$ . Table 4 shows the expected value of total wealth accrued to SG at different values of  $\gamma$ . In table 3 and 4, observe that the higher the parameter that measured the risk aversion coefficient  $\gamma$ , the higher the wealth that will accrued to the SG overtime. It implies that more the investor take risk, the higher the wealth, which is an intuitive result.

## 6. Conclusion

This paper examines the valuation and management of inflows of internally generated revenue and FG monetary allocations to the SG with the aim of determining the value of total wealth that will accrued to the SG over a period of time. The present as well as the terminal values of the SG total revenue were obtained. The SGs are advise to invest part of the internally generated revenue and FG monetary allocation into stocks and cash account to enhance maximum revenue for infrastructural development of the state.

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