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A BEHAVIORAL PORTFOLIO DECISION MODEL BASED ON CREDIBILITY OF INTERVAL-VALUED FUZZY NUMBER

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Abstract: This paper handles a multi-account behavioral portfolio decision problem involved with interval-valued fuzzy number return. A sentimental mean model for behavioral portfolio decision is proposed by taking into account multiple mental accounts and investor sentiments. The presented behavioral portfolio decision model maximizes the sentimental mean value of portfolio return and ensures the portfolio return of each mental account exceeding the given minimum interval level with a given credibility degree. Then, a behavioral portfolio decision model based on credibility degree of interval-valued fuzzy number return is designed to solve the optimal portfolio strategy. Finally, one numerical example is given to illustrate the validity of the proposed behavioral portfolio method.

Keywords: behavioral portfolio model; interval-valued fuzzy number; credibility degree; mental account.

2010 AMS Subject Classification: 91G10, 03E72.

1. INTRODUCTION

In 2000[1] Shefrin and Statman proposed behavioral portfolio theoretical framework for asset choice under uncertainty based on prospect theory [2,3]. In the behavioral portfolio process, each portfolio layer is associated with a separate mental account [4]. After that, Ma [5] proposed a practical decision-making method for behavioral portfolio choice. Mehlawat [6] and Amelia [7] developed multi-criteria behavioral portfolio decision models. Jin [8] developed multi-period and multi-objective behavioral portfolio selection approach. Also, Xie [9] studied the behavioral

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assets portfolio method based on sentiment recognition.

Recently, interval-valued fuzzy number (IvFN) has been generally used in handling and describing imprecise phenomena that often rise in financial and managerial systems. For example, Zhou [10] studied the Markov chain approximation to multi-stage interactive group decision-making method based on interval fuzzy number. Hasan Dalman [11] discussed the interactive goal programming algorithm with Taylor series and interval type 2 fuzzy numbers. Hu [12] discussed the Multi-criteria decision making method based on possibility degree of interval type-2 fuzzy number. In uncertain portfolio decision environment, the return of financial asset is conveniently evaluated by interval fuzzy number.

As far as we know, a lot of portfolio models based on interval and fuzzy number have been proposed to deal with portfolio decision problems with uncertain return and risk. For example, Bhattacharyya Rupak[13] studied Fuzzy mean–variance–skewness portfolio selection models by interval analysis. Xu [14] solved non-linear portfolio optimization problems with interval analysis, Liu [15], Zhang [16] discussed the multi-period portfolio selection optimization model by using interval analysis. Recently, Mohagheghi [17] studied the project portfolio selection by a new interval type-2 fuzzy optimization approach. Zhang [18] proposed some fuzzy portfolio models based on possibilistic mean and variance. Bhattacharyya Rupak [19] presented the portfolio selection model based on fuzzy entropy and skewness. Mehlawat [20], Yue [21] studied the fuzzy multi-objective higher order moment portfolio selection model. Liagkouras [22] and Liu [23] also discussed the fuzzy multi-period portfolio models. Jalota Hemant[24] presented a credibilistic decision support system for portfolio optimization. Liu [25] proposed the credibilistic multi-period portfolio optimization model with bankruptcy control and affine recourse. Zhang [26] studied portfolio adjusting optimization problem under credibility measures. Mehlawat [27] proposed a credibilistic mean-entropy models for multi-period portfolio selection with multi-choice aspiration levels.

However, the existing interval or fuzzy portfolio decision models cannot deal with the portfolio with interval-valued fuzzy number returns. And the investor’s sentiment and investor’s behavioral interacting factors have not been considered. Therefore, we will build a new behavioral portfolio decision framework with IvFNs for behavioral investors that follow ethical, environmental and social considerations in their investment process. To do so, we construct an interval-valued fuzzy behavioral portfolio theory with mental accounts and sentiment factors based on credibility of IvFNs. In this paper, we propose a credibilistic fuzzy behavioral portfolio model with IvFNs to determine the asset allocation between the different mental accounts according to the investor's sentiment and the historical interval fuzzy number return data in financial market.

2. PRELIMINARIES

Let us first review some basic concepts of interval-valued fuzzy numbers, which will be employed in the following sections about behavioral portfolio decision model with interval-valued fuzzy number return.

Definition 1[28]. An interval-valued fuzzy set $\tilde{R}$ is called an interval-valued fuzzy number (IvFN) if it satisfies the following properties: (1) $\tilde{R}$ is defined in a closed bounded interval of real line; (2) $\tilde{R}$ is a convex set. An IvFN $\tilde{R}$ can be expressed by the form $\tilde{R}=[\tilde{R}_L,\tilde{R}_U]=[(a_L,b_L,c_L,d_L),(a_U,b_U,c_U,d_U)]$, where $\tilde{R}_L,\tilde{R}_U$ are named as the lower fuzzy number and upper fuzzy number, respectively. Their membership functions satisfy the following forms, respectively.

$$\mu_{\tilde{R}_L}(x)=\begin{cases}(x-a_L)/(b_L-a_L), & if \quad a_L\le x<b_L,\\ 1\quad, & if \quad b_L\le x\le c_L,\\ (d_L-x)/(d_L-c_L), & if \quad c_L<x\le d_L,\\ 0\quad, & otherwise.\end{cases}$$

$$\mu_{\tilde{R}_U}(x)=\begin{cases}(x-a_U)/(b_U-a_U), & if \quad a_U\le x<b_U,\\ 1\quad, & if \quad b_U\le x\le c_U,\\ (d_U-x)/(d_U-c_U), & if \quad c_U<x\le d_U,\\ 0\quad, & otherwise.\end{cases}$$

Definition 2. Let $\tilde{R}=[\tilde{R}_L,\tilde{R}_U]=[(a_L,b_L,c_L,d_L),(a_U,b_U,c_U,d_U)]$ be an interval-valued fuzzy number, the λ-level cut set of lower fuzzy number $\tilde{R}_L$ and upper fuzzy number $\tilde{R}_U$ are respectively expressed as

$$R_L^\lambda=[b_L-\alpha_L(1-\lambda),c_L+\beta_L(1-\lambda)]=[a_L+(b_L-a_L)\lambda,d_L-(d_L-c_L)\lambda],\qquad \forall\lambda\in[0,1],$$

$$\tilde{R}_U^{\lambda} = [b_U - \alpha_U(1-\lambda), c_U + \beta_U(1-\lambda)] = [a_U + (b_U - a_U)\lambda, d_U - (d_U - c_U)\lambda], \quad \forall \lambda \in [0,1],$$

where $\alpha_L = (b_L - a_L)$, $\beta_L = (d_L - c_L)$ are left width and right width of lower fuzzy number $\tilde{R}_L$, respectively.

$\alpha_U = (b_U - a_U)$, $\beta_U = (d_U - c_U)$ are left width and right width of upper fuzzy number $\tilde{R}_U$, respectively.

Definition3. Let $\tilde{R}_1 = [(a_{1L}, b_{1L}, c_{1L}, d_{1L}), (a_{1U}, b_{1U}, c_{1U}, d_{1U})]$, $\tilde{R}_2 = [(a_{2L}, b_{2L}, c_{2L}, d_{2L}), (a_{2U}, b_{2U}, c_{2U}, d_{2U})]$ be two positive interval-valued fuzzy numbers, the addition and scale multiplication of IvFNs [28] are defined as follows.

(1) $\tilde{R}_1 + \tilde{R}_2 = [(a_{1L} + a_{2L}, b_{1L} + b_{2L}, c_{1L} + c_{2L}, d_{1L} + d_{2L}), (a_{1U} + a_{2U}, b_{1U} + b_{2U}, c_{1U} + c_{2U}, d_{1U} + d_{2U})$,

(2) $x\tilde{R}_1 = [(xa_{1L}, xb_{1L}, xc_{1L}, xd_{1L}), (xa_{1U}, xb_{1U}, xc_{1U}, xd_{1U})], \quad \forall x > 0$.

Definition 4 [18]. Let $\tilde{A} = (a, b, c, d)$ be a trapezoidal fuzzy number, the λ-level cut set of fuzzy number is defined as $\tilde{A}_\lambda = [b - \alpha (1-\lambda), c + \beta (1-\lambda)] = [a^-(\lambda), a^+(\lambda)], \quad \forall \lambda \in [0,1]$.

Theorem 1[18]. Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$, $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers, for all $\lambda \in [0,1]$, and positive real number $x > 0$, then we can get

(1) $(\tilde{A} + \tilde{B})_\lambda = \tilde{A}_\lambda + \tilde{B}_\lambda = [a^-(\lambda) + b^-(\lambda), a^+(\lambda) + b^+(\lambda)], \forall \lambda \in [0,1]$;

(2) $(x\tilde{A})_\lambda = x\tilde{A}_\lambda = [xa^-(\lambda), xa^+(\lambda)], \forall \lambda \in [0,1]$.

Definition 5. Let $\tilde{R} = [\tilde{R}_L, \tilde{R}_U] = [(a_L, b_L, c_L, d_L), (a_U, b_U, c_U, d_U)]$ be an IvFN, the lower and upper possibilistic mean value of interval-valued fuzzy number $\tilde{R}$ are, respectively, defined as

$$M_L(\tilde{R}) = \int_0^1 \lambda[R_L^-(\lambda) + R_L^+(\lambda)]d\lambda = \int_0^1 \lambda[(b_L + c_L) + (\beta_L - \alpha_L)(1-\lambda)]d\lambda = (b_L + c_L)/2 + (\beta_L - \alpha_L)/6;$$

$$M_U(\tilde{R}) = \int_0^1 \lambda[R_U^-(\lambda) + R_U^+(\lambda)]d\lambda = \int_0^1 \lambda[(b_U + c_U) + (\beta_U - \alpha_U)(1-\lambda)]d\lambda = (b_U + c_U)/2 + (\beta_U - \alpha_U)/6.$$

The possibilistic mean of IvFN $\tilde{R}$ can also be defined as

$$E(\tilde{R}) = (M_L(\tilde{R}) + M_U(\tilde{R}))/2 = (b_L + c_L + b_U + c_U)/4 + (\beta_L - \alpha_L)/12 + (\beta_U - \alpha_U)/12 . \quad (1)$$

Definition 6. Let $\tilde{R} = (a,b,c,d)$ be a fuzzy number with membership $\mu_{\tilde{R}}$, and r be a real number. The credibility of fuzzy number $\tilde{R}$ greater than r is defined as [27,29],

$$Cr(\tilde{R} \ge r) = [pos(\tilde{R} \ge r) + Nes(\tilde{R} \ge r)]/2 = [\sup_{x \ge r} \mu_{\tilde{R}}(x) + (1 - \sup_{x < r} \mu_{\tilde{R}}(x))]/2 \quad , \quad (2)$$

$$= \begin{cases} 1, & if \quad r \le a; \\ (2b - a - r)/2(b - a), & if \quad a < r \le b; \\ 1/2, & if \quad b < r \le c; \\ (d - r)/2(d - c), & if \quad c < r \le d; \\ 0, & if \quad r > d. \end{cases}$$

Definition 7. Let $\tilde{R} = [\tilde{R}_L, \tilde{R}_U] = [(a_L, b_L, c_L, d_L), (a_U, b_U, c_U, d_U)]$ be an interval-valued fuzzy number with membership $\mu_{\tilde{R}}$ and $\bar{r} = [r_1, r_2]$ be an interval, the credibility of fuzzy number $\tilde{R}$ greater than $\bar{r}$ is defined as

$$Cr(\tilde{R} \ge \bar{r}) = Cr([\tilde{R}_L, \tilde{R}_U] \ge [r_1, r_2]) = \min\{Cr(\tilde{R}_L \ge r_1),\ \ Cr(\tilde{R}_U \ge r_2)\} \quad . \quad (3)$$

Theorem 2[29]. Let $\tilde{R} = (a,b,c,d)$ be a fuzzy number, and r be a real number, then we can easily prove that

(1) If $Cr(\tilde{R} \ge r) \ge p$, for any $p > 1/2$, then $r \le (2p - 1)a + (2 - 2p)b$; (4)

(2) If $Cr(\tilde{R} \ge r) \ge p$, for any $p \le 1/2$, then $r \le 2pc + (1 - 2p)d$. (5)

Definition 8. Let s_{ij} be the sentiment of investor on asset j at mental account i, $\tilde{R}_{ij}$ be the interval-valued fuzzy number return of asset j in mental account i, the sentiment influential function $f(s_{ij})$ and the sentiment-adjusted interval-valued fuzzy number return $\tilde{R}_{ij}$

are respectively defined as [9],

$$f(s_{ij}) = e^{\kappa s_{ij}}, (\kappa > 0), \qquad \tilde{\hat{R}}_{ij} = f(s_{ij})\tilde{R}_{ij} = e^{\kappa s_{ij}} \tilde{R}_{ij}. \qquad (6)$$

Obviously, $f(s_{ij}) > 0$ and is a increasing function ,that is to say, the higher the investor's sentiment becomes, the greater the estimated return of security becomes. In fact, if sentiment s_{ij} is positive, investor is optimistic on asset j of mental account i, the adjusted return of asset will become higher ; if sentiment s_{ij} is negative, investor is pessimistic on asset j of mental account i ,the adjusted return of asset will become less .

3. Formulation of Behavioral Portfolio Decision Problem with IvFN Returns

In this part, we discuss the behavioral portfolio decision problem with interval-valued fuzzy number returns and investor sentiments. We first introduce the problem description and notations used in the following sections. Then, we formulate the interval-valued fuzzy behavioral portfolio decision model by maximizing the sentiment-adjusted mean of portfolio return with IvFNs.

3.1. Basic notations of Behavioral Portfolio Decision Problem

Let us consider a behavioral portfolio decision problem with m mental accounts. Suppose mental account i consists n_i risky assets. The return rates of risky assets are evaluated by interval-valued fuzzy numbers. Assume that the investor intends to allocate his/her wealth among the n_i $(i = 1, 2, \cdots, m)$ risky securities for making accounting investment plan in m mental accounts. For convenience, we put together all the notations below.

x_{ij} : the investment proportion of risky security j in mental account i ;

l_{ij} : the lower boundary of investment proportion of risky security j in mental account i ;

u_{ij} : the upper boundary of investment proportion of risky security j in mental account i ;

ω_i : the importance degree of the holding mental account i ;

s_{ij} : the sentiment of investor on security j in mental account i ;

$f(s_{ij})$: the sentiment influential function of investor on security j in mental account i .

3.2. Sentimental-mean of interval-valued fuzzy number return for portfolio

Assume that the whole investment process is self-financing, that is, the investor does not invest the additional capital during the portfolio decision process. Let $\tilde{R}_{ij}=[\tilde{R}_{ijL},\tilde{R}_{ijU}]=[(a_{ijL},b_{ijL},c_{ijL},d_{ijL}),(a_{ijU},b_{ijU},c_{ijU},d_{ijU})]$ be the interval-valued fuzzy number return of asset j in mental account i. According to the previous section, the sentimental mean value of interval-valued fuzzy number return for portfolio $x_i=(x_{i1},x_{i2},\cdots,x_{in_i})$ in mental account i $(i=1,2,\cdots,m)$ is determined by

$$E(\tilde{R}_i)=\sum_{j=1}^{n_i}x_{ij}E(\tilde{R}_{ij})f(s_{ij})$$

$$=\sum_{j=1}^{n_i}x_{ij}[(b_{ijL}+c_{ijL}+b_{ijU}+c_{ijU})/4+(\beta_{ijL}-\alpha_{ijL})/12+(\beta_{ijU}-\alpha_{ijU})/12]f(s_{ij}).$$

3.3. Multi-account Behavioral portfolio decision model based on credibility degree of IvFN return

Assume there are m mental accounts for security market and the investor wants to maximize the sentiment-adjusted mean of portfolio return over the whole m mental accounts. Meanwhile, the investor hopes that the interval-valued fuzzy portfolio return at each mental account must achieve or exceed the given minimum interval number level with a credibility degree. Thus, the interval-valued fuzzy behavioral portfolio decision selection problem with multi-accounts can be formulated as the following programming model denoted by

$$\text{(P1)}\quad \max \sum_{i=1}^{m}\omega_i\Big(\sum_{j=1}^{n_i}x_{ij}E(\tilde{R}_{ij})f(s_{ij})\Big)-\sum_{i=1}^{m}\sum_{j=1}^{n_i}hx_{ij}$$

$$\text{s.t.}\quad Cr\Big(\sum_{j=1}^{n_i}x_{ij}\tilde{R}_{ij}\ge\bar{r}_i=[r_{i1},r_{i2}]\Big)\ge p_i,\quad i=1,2,\cdots,m.$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n_i}x_{ij}=1,$$

$$0\le l_{ij}\le x_{ij}\le u_{ij}\le 1\ ,\qquad i=1,2,\cdots,m;\ j=1,2,\cdots,n_i;$$

where $\omega=(\omega_1,\omega_2,\cdots,\omega_m)$ is the weight vector of all the mental accounts for security market investor, $\sum_{i=1}^{m}\omega_i=1,\omega_i\in[0,1]$, ω_i is the importance degree of mental account i. h is assumed as the unit transaction cost of each security in all the mental accounts. And $\bar{r}_i=[r_{i1},r_{i2}]$ represents the given minimum aspiration interval return level of the portfolio regarding mental account i; p_i is the given credibility degree assuring that the interval-valued fuzzy number return of mental account i greater than the given minimum aspiration interval return level $\bar{r}_i$. In general, the lower level the mental account i is, the greater the parameter p_i is.

In this article, we assume the investment sentiment function on asset j of mental account i is $f(s_{ij}) = e^{0.3 s_{ij}}$. Then the above programming model (P1) can be transformed to the following optimization model (P_1') according to Definition 7 of credibility degree of interval-valued fuzzy number returns and Theorem 2.

(P_1')

$$\max \sum_{i=1}^{m} \omega_i \sum_{j=1}^{n_i} x_{ij}[(b_{ijL} + c_{ijL} + b_{ijU} + c_{ijU})/4 + (\beta_{ijL} - \alpha_{ijL})/12 + (\beta_{ijU} - \alpha_{ijU})/12] f(s_{ij}) - \sum_{i=1}^{m} \sum_{j=1}^{n_i} h x_{ij}$$

s.t.

$$\begin{cases} Cr(\sum_{j=1}^{n_i} x_{ij} \tilde{R}_{ijL} \geq r_{i1}) \geq p_i, \quad i = 1,2,\cdots,m \\ Cr(\sum_{j=1}^{n_i} x_{ij} \tilde{R}_{ijU} \geq r_{i2}) \geq p_i, \quad i = 1,2,\cdots,m \\ \sum_{i=1}^{m} \sum_{j=1}^{n_i} x_{ij} = 1, \\ 0 \leq l_{ij} \leq x_{ij} \leq u_{ij} \leq 1, \quad i = 1,2,\cdots,m; \quad j = 1,2,\cdots,n_i, \end{cases}$$

The above model is equivalent to the following programming model.

(P_1'')

$$\max \sum_{i=1}^{m} \omega_i \sum_{j=1}^{n_i} x_{ij}[(b_{ijL} + c_{ijL} + b_{ijU} + c_{ijU})/4 + (\beta_{ijL} - \alpha_{ijL})/12 + (\beta_{ijU} - \alpha_{ijU})/12] f(s_{ij}) - \sum_{i=1}^{m} \sum_{j=1}^{n_i} h x_{ij}$$

s.t.

$$\begin{cases} Cr(\sum_{j=1}^{n_i} x_{ij}(a_{ijL}, b_{ijL}, c_{ijL}, d_{ijL}) \geq r_{i1}) \geq p_i, \quad i = 1,2,\cdots,m \\ Cr(\sum_{j=1}^{n_i} x_{ij}(a_{ijU}, b_{ijU}, c_{ijU}, d_{ijU}) \geq r_{i2}) \geq p_i, \quad i = 1,2,\cdots,m \\ \sum_{i=1}^{m} \sum_{j=1}^{n_i} x_{ij} = 1, \\ 0 \leq l_{ij} \leq x_{ij} \leq u_{ij} \leq 1, \quad i = 1,2,\cdots,m; \quad j = 1,2,\cdots,n_i. \end{cases}$$

4. NUMERICAL EXAMPLE

In order to illustrate the detailed decision process of our model and the effectiveness of the proposed behavioral portfolio method, we give a numerical example for simulating the real security investment transaction below. For simplicity, in the following example we only consider two-mental accounting behavioral portfolio decision problem with interval-valued fuzzy number returns.

Example 1. Assume the security market investor has two mental accounts MA_1, MA_2. The lower-level mental account MA_1 includes three alternative securities A_{11}, A_{12}. The high-level mental account MA_2 includes three alternative securities A_{21}, A_{22}. All the financial securities in the above two mental accounts are selected from Shenzhen Stock Exchange in China. To simulate the transaction, we collect the weekly closing pricing of assets from Dec 2018 to Dec 2019, with 1 yearly observations. By analyzing the stock historical data, the corporation financial reports and future information, we can employ the statistical percentile method [30] to assess the interval-valued fuzzy number return $\tilde{\tilde{R}}_{ij} = [\tilde{\tilde{R}}_{ijL}, \tilde{\tilde{R}}_{ijU}] = [(a_{ijL}, b_{ijL}, c_{ijL}, d_{ijL}), (a_{ijU}, b_{ijU}, c_{ijU}, d_{ijU})]$ of securities A_{ij} $(i = 1,2;\ j = 1,2)$ in the above two mental accounts. The evaluated interval-valued fuzzy number return of the selected stocks are listed in the following Table 1.

Table 1. The assessed interval-valued fuzzy number returns of the selected securities from two mental accounts

Mental account 1	**Interval-valued Fuzzy number return**
Security A_{11}	([0.04, 0.08, 0.10, 0.18], [0.02, 0.07, 0.15, 0.2])
Security A_{12}	([0.03, 0.06, 0.08, 0.13], [0.01, 0.05, 0.10, 0.15])
Mental account 2	**Interval-valued Fuzzy number return**
Security A_{21}	([0.1, 0.2, 0.3, 0.32], [0.08, 0.15, 0.35, 0.38])
Security A_{22}	([0.15, 0.25, 0.28, 0.3], [0.1, 0.2, 0.4, 0.42])

In this example, we assume $r_1=[r_{11},r_{12}]=[0.023,0.085]$, $r_2=[r_{21},r_{22}]=[0.2,0.3]$ are the given minimum expected interval return of the portfolio for mental account 1 and 2, respectively. And we let $p_1=0.89$ be the possibility degree that the interval-valued fuzzy number return of the first mental account MA_1 exceeds the minimum interval $r_1=[0.023,0.085]$, and $p_2=0.04$ is the given possibility degree that the interval-valued fuzzy return of the second mental account MA_2 exceeds the minimum aspiration $r_2=[0.2,0.3]$.

In order to obtain the optimal portfolio strategy $x=(x_{11},x_{12},x_{21},x_{22})$, we construct the following interval-valued fuzzy behavioral portfolio decision model (P2).

(P2) $\max \sum_{i=1}^{2}\omega_i\sum_{j=1}^{2}x_{ij}E(\tilde{R}_{ij})f(s_{ij})-\sum_{i=1}^{2}\sum_{j=1}^{2}hx_{ij}$

$$\text{s.t.}\quad \begin{cases} r_{11}\le(2p_1-1)\sum_{j=1}^{2}x_{1j}a_{1jL}+2(1-p_1)\sum_{j=1}^{2}x_{1j}b_{1jL} \\ r_{12}\le(2p_1-1)\sum_{j=1}^{2}x_{1j}a_{1jU}+2(1-p_1)\sum_{j=1}^{2}x_{1j}b_{1jU} \\ r_{21}\le 2p_2\sum_{j=1}^{2}x_{2j}c_{2jL}+(1-2p_2)\sum_{j=1}^{2}x_{2j}d_{2jL} \\ r_{22}\le 2p_2\sum_{j=1}^{2}x_{2j}c_{2jU}+(1-2p_2)\sum_{j=1}^{2}x_{2j}d_{2jU} \\ \sum_{i=1}^{2}\sum_{j=1}^{2}x_{ij}=1, \\ 0\le l_{ij}\le x_{ij}\le u_{ij}\le 1,\quad i=1,2;\quad j=1,2. \end{cases}$$

Since $p_1=0.89>1/2$, $p_2=0.04<1/2$, we can transform the above interval-valued fuzzy behavioral portfolio decision model to the following linear programming model **(P3)** by employing Theorem 2.

(P3) $\max \sum_{i=1}^{2}\omega_i\sum_{j=1}^{2}x_{ij}E(\tilde{R}_{ij})f(s_{ij})-\sum_{i=1}^{2}\sum_{j=1}^{2}hx_{ij}$

$$\text{s.t.}\quad \begin{cases} r_{11}\le(2p_1-1)\sum_{j=1}^{2}x_{1j}a_{1jL}+2(1-p_1)\sum_{j=1}^{2}b_{1jL} \\ r_{12}\le(2p_1-1)\sum_{j=1}^{2}x_{1j}a_{1jU}+2(1-p_1)\sum_{j=1}^{2}b_{1jU} \\ r_{21}\le 2p_2\sum_{j=1}^{2}x_{2j}c_{2jL}+(1-2p_2)\sum_{j=1}^{2}x_{2j}d_{2jL} \\ r_{22}\le 2p_2\sum_{j=1}^{2}x_{2j}c_{2jU}+(1-2p_2)\sum_{j=1}^{2}x_{2j}d_{2jU} \\ \sum_{i=1}^{2}\sum_{j=1}^{2}x_{ij}=1, \\ 0\le l_{ij}\le x_{ij}\le u_{ij}\le 1,\quad i=1,2;\quad j=1,2. \end{cases}$$

By utilizing the following possibilistic mean formula (1) of interval-valued fuzzy number,

$$E(\tilde{R}_{ij}) = [b_{ijL} + c_{ijL} + b_{ijU} + c_{ijU}]/4 + [(\beta_{ijL} - \alpha_{ijL}) + (\beta_{ijU} - \alpha_{ijU})]/12,$$

and substituting data $\tilde{R}_{ij} = [\tilde{R}_{ijL}, \tilde{R}_{ijU}] = [(a_{ijL}, b_{ijL}, c_{ijL}, d_{ijL}), (a_{ijU}, b_{ijU}, c_{ijU}, d_{ijU})]$ of interval-valued fuzzy number return of security A_{ij} in Table 1, we can compute the possibilistic mean vector of the selected stock securities as

$$E(\tilde{R}_{11}) = 0.1033, \quad E(\tilde{R}_{12}) = 0.075, \quad E(\tilde{R}_{21}) = 0.24, \quad E(\tilde{R}_{22}) = 0.2692.$$

Suppose that the investor's initial sentiment vector on the selected four securities is

$$s = (s_{11}, s_{12}, s_{21}, s_{22}) = (-0.2, 0, 1, 2).$$

Then we choose $f(s) = e^{0.3s}$ as the sentiment influential function and compute the sentiment influential function value vector as

$$f(s) = (f(s_{11}), f(s_{12}), f(s_{21}), f(s_{22})) = (e^{0.3s_{11}}, e^{0.3s_{12}}, e^{0.3s_{21}}, e^{s_{22}}) = (0.9418,\ 1,\ 1.3499,\ 1.8221).$$

In this example, we also assume the lower boundary l_{ij} and upper boundary u_{ij} of investment proportion of risky security j at mental account i are 0.01 and 0.5, respectively. Suppose h =0.0053 is the transaction cost of each security in all the mental accounts.

Hence, the above model (P3) can be transformed to the following model (P4).

(P4) $$\max\ \omega_1(0.0973x_{11} + 0.075x_{12}) + \omega_2(0.324x_{21} + 0.4905x_{22}) - \sum_{i=1}^{2} \sum_{j=1}^{2} 0.0053x_{ij}$$

$$\text{s.t.} \begin{cases} 0.023 \le 0.78(0.04x_{11} + 0.03x_{12}) + 0.22(0.08x_{11} + 0.06x_{12}) \\ 0.085 \le 0.78(0.02x_{11} + 0.01x_{12}) + 0.22(0.07x_{11} + 0.05x_{12}) \\ 0.2 \le 0.08(0.3x_{21} + 0.28x_{22}) + 0.92(0.32x_{21} + 0.3x_{22}) \\ 0.3 \le 0.08(0.35x_{21} + 0.4x_{22}) + 0.92(0.38x_{21} + 0.42x_{22}) \\ x_{11} + x_{12} + x_{21} + x_{22} = 1 \\ 0.01 \le x_{ij} \le 0.5, \quad i = 1,2; \quad j = 1,2 \end{cases}$$

In fact, the above model (P4) is equivalent to the following linear programming models (P5-P6).

(P5) $$\max \omega_1(0.0973x_{11}+0.075x_{12})+\omega_2(0.324x_{21}+0.4905x_{22})-\sum_{i=1}^{2}\sum_{j=1}^{2}0.0053x_{ij}$$

$$\text{s.t.}\quad \begin{cases} 0.023\le 0.0312x_{11}+0.0234x_{12}+0.0176x_{11}+0.0132x_{12} \\ 0.085\le 0.0156x_{11}+0.0078x_{12}+0.0154x_{11}+0.011x_{12} \\ 0.2\le 0.024x_{21}+0.0224x_{22}+0.2944x_{21}+0.276x_{22} \\ 0.3\le 0.028x_{21}+0.032x_{22}+0.3496x_{21}+0.3864x_{22} \\ x_{11}+x_{12}+x_{21}+x_{22}=1 \\ 0.01\le x_{ij}\le 0.5,\quad i=1,2;\quad j=1,2 \end{cases}$$

(P6) $$\max \omega_1(0.0973x_{11}+0.075x_{12})+\omega_2(0.324x_{21}+0.4905x_{22})-\sum_{i=1}^{2}\sum_{j=1}^{2}0.0053x_{ij}$$

$$\text{s.t.}\quad \begin{cases} 0.023\le 0.0488x_{11}+0.0366x_{12} \\ 0.085\le 0.031x_{11}+0.0188x_{12} \\ 0.2\le 0.3184x_{21}+0.2984x_{22} \\ 0.3\le 0.3776x_{21}+0.4184x_{22} \\ x_{11}+x_{12}+x_{21}+x_{22}=1 \\ 0.01=l_{ij}\le x_{ij}\le u_{ij}=0.5, i=1,2;\ j=1,2 \end{cases}$$

If we consider the weight vector of two mental accounts $\omega=(\omega_1,\omega_2)=(0.1,0.9)$, the above behavioral portfolio model is transformed to the following linear programming model.

$$\max 0.1\times(0.0973x_{11}+0.075x_{12})+0.9\times(0.324x_{21}+0.4905x_{22})-\sum_{i=1}^{2}\sum_{j=1}^{2}0.0053x_{ij}$$

$$\text{s.t.}\quad \begin{cases} -0.0488x_{11}-0.0366x_{12}\le -0.023 \\ -0.031x_{11}-0.0188x_{12}\le -0.085 \\ -0.3184x_{21}-0.2984x_{22}\le -0.2 \\ -0.3776x_{21}-0.4184x_{22}\le -0.3 \\ x_{11}+x_{12}+x_{21}+x_{22}=1 \\ 0.01\le x_{ij}\le 0.5,\quad i=1,2;\quad j=1,2. \end{cases}$$

Since the different importance agree of each mental account affects the behavioral portfolio solution, in this paper we shall consider four kinds of investment importance vectors W1=(0.1,0.9) , W2=(0.45,0.55), W3=(0.85,0.15), W4=(0.9,0.1) of the two mental accounts and compare the optimal strategy for the interval-valued fuzzy behavioral portfolio decision model.

By utilizing the optimization toolbox in Matlab software we solve the above-mentioned linear programming models with respect to different weight vector of two mental accounts. Finally, we obtain the optimal behavioral portfolio strategy, which is the solver corresponding to maximum sentiment-adjusted mean objective function value of behavioral portfolio. The optimal behavioral portfolio solution $x^* = (x_{11}^*, x_{12}^*, x_{21}^*, x_{22}^*)$ regarding the different weight vector of mental accounts are computed and listed in the following Table 2.

Table 2. Optimal behavioral portfolio result with respect to different weight vector of two mental accounts

Weight vector of Mental accounts	**Maximum expected net return of portfolio**	x_{11}^*	x_{12}^*	x_{21}^*	x_{22}^*
W=(0.1,0.9)	**0.1951**	0.4232	0.1	0.1	0.3768
W=(0.45,0.55)	**0.1361**	0.423	0.1	0.1	0.377
W= (0.85,0.15)	**0.0694**	0.5	0.1	0.1	0.3
W=(0.9,0.1)	**0.0638**	0.5000	0.1317	0.1000	0.2683

5. CONCLUSION

In this article, we consider the multi-account behavioral portfolio decision problem under interval-valued fuzzy environment. We use the sentiment-adjusted mean to measure trapezoidal interval-valued fuzzy number return of the behavioral portfolio. Furthermore, based on the credibility degree of interval-valued fuzzy number return of each mental account exceeding the given minimum interval level we present a new behavioral portfolio model with interval-valued fuzzy return and investor’s sentiment. In order to solve the proposed interval-valued fuzzy behavioral model, we transform it into the equivalent programming models. Finally, a numerical example is given to illustrate the effectiveness of the proposed behavioral portfolio decision method.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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