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AN INVENTORY MODEL WITH QUADRATIC DEMAND PATTERN AND DETERIORATION WITH SHORTAGES UNDER THE INFLUENCE OF INFLATION

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Abstract: The objective of this model is to investigate the inventory system for perishable items with quadratic demand pattern where time-dependent deterioration is considered. The Economic order quantity is determined for minimizing the average total cost per unit time under the influence of inflation and time value of money. The influences of inflation and time-value of money on the inventory system are investigated with the help of some numerical examples.

Keywords: Inventory system, Quadratic demand, Deterioration, Shortage.

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1. Introduction

The deterioration of many items during storage period is a real fact. In many inventory models, it is assumed that the items can be stored indefinitely without any

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risk of deterioration. However, certain types of items undergo changes while in storage so that, with time, they become partially or entirely unfit for use. Deterioration in general may be considered as the result of various affect on the stock, some of which have damage, spoilage, decay, decreasing, usefulness, dryness, evaporation and many more. There are several types of items that will deteriorate if stored for extended periods of time. Examples of deteriorating items include metal parts, which are prone to corrosion and rusting, and food items, which are subject to spoilage and decay. Electronic components and fashion clothing also fall into this category, because they can become obsolete over time and their demand will typically decrease drastically.

Inventory control for deteriorating items has been a well-studied problem. Numerous optimal and heuristic approaches have been developed for modeling and solving different variations of this problem. C.K.Tripathy, L.M.Pradhan and U.Mishra[23] have developed a model with linear deterioration rate where the demand rate is considered to be constant. Sarbjit and S.S.raj[18] have developed a model whose demand as well as perishability rate increases with time. Ghare and Schrader [11] use the concept of deterioration. Covert and Philip [5] also formulated a model with variable rate of deterioration with two parameter Weibull distributions which was further extended by Shah [20]. Sarbjit and Raj [19] also developed the model for items having linear demand and variable Deterioration rate with trade credit. Tomba et.al[22] developed a model with linear demand pattern and deterioration with shortages. Sanjay Jain and Kumar [13] established the model having power demand pattern, Weibull distribution deterioration and shortages.

Within [24] considered fashion goods deteriorating at the end of a prescribed shortage period. Ghare and Schrader [11] develop a model for an exponentially decaying inventory. Various types of order-level inventory models for items deteriorating at a constant were discussed by Shah and Jaiswal [21], Aggarwal [1], Dave and Patel[8], Roychowdhury and Choudhuri [16], Dave [7] and Bahari Kashani [2], Inventory models with a time-depending rate of deterioration were studied by

Covert and Philip [5], Philip [15], Mishra [14], and Deb and Chaudhury [9]. In the classical inventory models, the demand rate is assumed to be a constant. In reality, demand for physical goods may be time dependent, stock dependent and price dependent. The first analytic model for linearly time-dependent demand was developed by Donaldson [10]. Most of these papers take the replenishment rate to be infinite.

In last two decades the economic situation of most countries have changed to an extent due to sharp decline in the purchasing power of money, that it has not been possible to ignore the effects of time value of money. Data and Pal [6], Bose et al. [3] have developed the EOQ model incorporating the effects of time value of money, a linear time dependent demand rate. Further Sana [17] considered the money value and inflation in a new level. Wee and Law (1999) [25] addressed the problem with finite replenishment rate of deteriorating items taking account of time value of money. Chang (2004) [4] proposes an inventory model for deteriorating items under inflation under a situation in which the supplier provides the purchaser a permissible delay in of payment if the purchaser orders a large quantity. Jaggi et al. (2006) [12] developed an inventory model for deteriorating items with inflation induced demand under fully backlogged demand.

In this paper an attempt has been made to develop an inventory model for perishable items with time proportional deterioration rate and the quadratic demand pattern is used over a finite planning horizon. Nature of the model is also discussed for shortage state. We consider the time-vale of money and inflation of each cost parameter .Optimal solution for the proposed model is derived and the applications are investigated with the help of some numerical examples.

2. Assumptions and Notations

Following assumptions are made for the proposal model:

i. Single inventory will be used.

ii. Lead time is zero.

iii. The model is studied when shortages are allowed.

iv. Demand follows the quadratic demand pattern.

v. The deterioration rate is time proportional.

vi. Shortages are allowed and are completely backlogged.

vii. Replenishment rate is infinite but size is finite.

viii. Time horizon is finite.

ix. There is no repair of deteriorated items occurring during the cycle.

Following notations are made for the given model:

$I(t)$ = On hand inventory level at any time t , $t \geq 0$.

$R(t) = a + bt + c\,t^2$ is the quadratic demand rate at any time

$t \geq 0, a \geq 0, b > 0, c > 0$.

$\theta : \theta = \alpha\, t$, is the time-proportional deterioration rate. Where $0 < \alpha << 1, t \geq 0.$

Q = Total amount of replenishment in the beginning of each cycle.

V = Inventory at time $t = 0$

T = Duration of a cycle.

c_d = The deterioration cost per unit item.

c_h = The holding cost per unit item.

c_b = The shortage cost per unit.

U =The total average cost of the system.

3. Formulation

The objective of the model is to determine the optimal order quantity in order to keep the total relevant cost as low as possible. The optimality is determined for shortage of items. Taking Q be the total amount of replenishment in the beginning of each cycle, and after fulfilling backorders let V be the level of initial inventory. In the period

$(0,t_1)$ the inventory level gradually decreases due to market demand and deterioration.

At t_1, the level of inventory reaches zero and after that the shortages are allowed to occur during the interval $[t_1,T]$, which are fully backlogged. Only the backlogging items are replaced by the next replenishment.

Here we have taken the total duration T as fixed constant. The objective here is to determine the optimal order quantity in order to keep the total relevant cost as low as possible.

If $I(t)$ be the on-hand inventory at time $t \geq 0$, then at time $t+\Delta t$, the on-hand inventory in the interval $[0,t_1]$ will be

$$I(t+\Delta t) = I(t) - \theta(t)\, I(t)\, \Delta t - R(t)\, \Delta t$$

Dividing by Δt and then taking as $\Delta t \to 0$ we get

(3.1) $$\frac{dI(t)}{dt} + \alpha\, t\, I(t) = -(a+bt+c\,t^2)\,; \quad 0 \leq t \leq t_1$$

For the next interval $[t_1,T]$, where the shortages are allowed we have

$$I(t+\Delta t) = I(t) - R(t)\cdot\Delta t\,.$$

Dividing by Δt and then taking as $\Delta t \to 0$ we get

(3.2) $$\frac{dI(t)}{dt} = -(a+bt+c\,t^2); \qquad t_1 \leq t \leq T$$

The boundary conditions are $I(0)=V$ and $I(t_1)=0$.

On solving equation (3.1) with boundary condition we have

(3.3)

$$I(t) = e^{\frac{-\alpha t^2}{2}}\left[a(t_1-t) + \frac{b}{2}(t_1^2-t^2) + \frac{2c+a\alpha}{6}(t_1^3-t^3) + \frac{b\,\alpha}{8}(t_1^4-t^4) + \frac{c\,\alpha}{10}(t_1^4-t^4)\right];$$

$0 \le t \le t_1$

On solving equation (3.2) with boundary condition we have

(3.4) $$I(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3)$$

; $t_1 \le t \le T$

Using $I(0) = V$ in equation (3.3) we have

(3.5) $$V = at_1 + \frac{b}{2}t_1^2 + \frac{2c + a\alpha}{6}t_1^3 + \frac{b\,\alpha}{8}t_1^4 + \frac{c\,\alpha}{10}t_1^5$$

The total amount of deteriorated units in $0 \le t \le t_1$ is given by

(3.6) $$\int_0^{t_1} \alpha\, tI(t)\, dt$$

$$= \alpha\left[\frac{1}{2}\left\{\frac{2a}{3}t_1^3 + \frac{5b}{8}t_1^4 + \frac{12c + 5a\alpha}{30}t_1^5 + \frac{b\,\alpha}{8}t_1^6 + \frac{c\,\alpha}{10}t_1^7\right\}\right]$$

The total cost function consists of the following elements:

(i) Holding cost per cycle

(3.7) $$C_h \int_0^{t_1} I(t)dt$$

$$= C_h\left[t_1^2\,\frac{a}{2} + t_1^3\left\{\frac{b}{3} + \frac{(i-r)a}{6}\right\} + t_1^4\left\{\frac{c}{4} - \frac{a\alpha}{12} + \frac{(i-r)b}{8}\right\} + t_1^5\left\{\frac{(i-r)(c-a\alpha)}{10} - \frac{b\alpha}{60}\right\}\right.$$

$$+ t_1^6\left\{\frac{1-c\alpha}{90} - \frac{(i-r)b\alpha}{24}\right\} + t_1^7\left\{\frac{(i-r)}{105} - \frac{(i-r)c\alpha}{30} - \frac{(i-r)a\alpha^2 + b\alpha^2}{48}\right\}$$

$$\left. + t_1^8\left\{\frac{c\alpha}{10} - \frac{(i-r)b\alpha^2}{64}\right\} + t_1^9\,\frac{(i-r)c\alpha^2}{80}\right]$$

(ii) Deterioration cost per cycle

(3.8) $C_d\int_0^{t_1}\alpha\beta\, t^{\beta-1}I(t)\,dt$

$$=\alpha\, C_d\left[\frac{a\,t_1^3}{6}+t_1^4\left\{\frac{3b}{8}+\frac{(i-r)a}{12}\right\}+t_1^5\left\{\frac{7(2c+a\alpha)}{60}+\frac{4(i-r)b}{15}\right\}+t_1^6\left\{\frac{b\alpha}{16}+\frac{(2c+a\alpha)(i-r)}{12}\right\}\right.$$

$$\left.+t_1^7\left\{\frac{c\alpha}{20}+\frac{b\alpha(i-r)}{24}\right\}+t_1^8\frac{c\alpha(i-r)}{30}\right]$$

(iii) Shortage cost per cycle

(3.9) $C_b\int_{t_1}^{T}I(t)dt$

$$=C_b\left[t_1\left\{\frac{2aT+a(i-r)T^2}{2}\right\}+t_1^2\left\{\frac{2bT+b(i-r)T^2-2a}{4}\right\}+t_1^3\left\{\frac{2cT-b+c(i-r)T^2-a(i-r)}{6}\right\}\right.$$

$$\left.-t_1^4\left\{\frac{2c+b(i-r)}{8}\right\}-t_1^5\frac{c(i-r)}{10}-\frac{aT^2}{2}-\frac{bT^3}{3}-\frac{cT^4}{12}-\frac{a(i-r)T^3}{3}-\frac{b(i-r)T^4}{8}-\frac{c(i-r)T^5}{15}\right]$$

Taking the relevant costs mentioned above, the total average cost per unit time of the system is given by

(3.10) $U(t_1)=\dfrac{1}{T}$ {Holding cost + Deterioration cost - Shortage cost}

$$=\frac{1}{T}\left[C_h\left[t_1^2\frac{a}{2}+t_1^3\left\{\frac{b}{3}+\frac{(i-r)a}{6}\right\}+t_1^4\left\{\frac{c}{4}-\frac{a\alpha}{12}+\frac{(i-r)b}{8}\right\}+t_1^5\left\{\frac{(i-r)(c-a\alpha)}{10}-\frac{b\alpha}{60}\right\}\right.\right.$$

$$+t_1^6\left\{\frac{1-c\alpha}{90}-\frac{(i-r)b\alpha}{24}\right\}+t_1^7\left\{\frac{(i-r)}{105}-\frac{(i-r)c\alpha}{30}-\frac{(i-r)a\alpha^2+b\alpha^2}{48}\right\}$$

$$+t_1^8\left\{\frac{c\alpha}{10}-\frac{(i-r)b\alpha^2}{64}\right\}+t_1^9\,\frac{(i-r)c\alpha^2}{80}\Bigg]$$

$$+\alpha\, C_d\left[\frac{a\,t_1^3}{6}+t_1^4\left\{\frac{3b}{8}+\frac{(i-r)a}{12}\right\}+t_1^5\left\{\frac{7(2c+a\alpha)}{60}+\frac{4(i-r)b}{15}\right\}+t_1^6\left\{\frac{b\alpha}{16}+\frac{(2c+a\alpha)(i-r)}{12}\right\}\right.$$

$$\left.+t_1^7\left\{\frac{c\alpha}{20}+\frac{b\alpha(i-r)}{24}\right\}+t_1^8\frac{c\alpha(i-r)}{30}\right]$$

$$-C_b\left[t_1\left\{\frac{2aT+a(i-r)T^2}{2}\right\}+t_1^2\left\{\frac{2bT+b(i-r)T^2-2a}{4}\right\}+t_1^3\left\{\frac{2cT-b+c(i-r)T^2-a(i-r)}{6}\right\}\right.$$

$$\left.-t_1^4\left\{\frac{2c+b(i-r)}{8}\right\}-t_1^5\,\frac{c(i-r)}{10}-\frac{aT^2}{2}-\frac{bT^3}{3}-\frac{cT^4}{12}-\frac{a(i-r)T^3}{3}-\frac{b(i-r)T^4}{8}-\frac{c(i-r)T^5}{15}\right]\Bigg]$$

Now equation (3.10) can be minimized but as it is difficult to solve the problem by deriving a closed equation of the solution of equation (3.10), Matlab Software has been used to determine optimal t_1^* and hence the optimal cost $U(t_1^*)$ can be evaluated. Also level of initial inventory level V^* in the beginning of each cycle can be determined.

4. Example

Example- 1:

The values of the parameters are considered as follows:

$c_h = \$40/unit/year, c_d = \$70/unit,\ c_b = \$100/unit$

$\alpha = 0.01,\ a = 0.24, b = 0.12, c = 0.04, T = 1\,year, i = 0.05, r = 0.12.$

Now using equation **(3.10)** which can be minimized to determine optimal

$t_1^* = 0.7827\, Years$ and hence the average optimal cost $U(t_1^*) = \$5.6661/unit$.

Also level of initial inventory level $V^* = 0.2313\, units$.

Example- 2:

The values of the parameters are considered as follows:

$c_h = \$40/unit/year, c_d = \$70/unit, c_b = \$100/unit$

$\alpha = 0.01,\ a = 0.5, b = 0.12, c = 0.04, T = 1\, year, i = 0.05, r = 0.12.$

Now using equation **(3.10)** which can be minimized to determine optimal

$t_1^* = 0.7387\, Years$ and hence the average optimal cost $U(t_1^*) = \$9.4055/unit$.

Also level of initial inventory level $V^* = 0.4079\, units$.

5. Conclusion

Here an EOQ model is derived for perishable items with quadratic demand pattern. Time-proportional deterioration rate is used. The model is studied for minimization of total average cost. Numerical examples are used to illustrate the result where we saw that when the value of a increases, the optimal time decreases whereas initial inventory level V^* increases.

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