Available online at http://scik.org
Math. Finance Lett. 2023, 2023:2
https://doi.org/10.28919/mfl/8258
ISSN: 2051-2929

NONPARAMETRIC ESTIMATION OF SOME DIVIDEND PROBLEMS BY FOURIER SINC SERIES EXPANSION IN THE WIENER-POISSON RISK MODEL

MARCELIN ROMEO NOUMEGNI KENMOE[1,*], JANE AKINYI ADUDA[2], MBELE BIDIMA MARTIN LE DOUX[3]

[1]Department of Mathematics, Pan African University Institute for Basic Sciences Technology and Innovation (PAUSTI), Nairobi, Kenya

[2]Department of statistics and actuarial sciences, Jomo Kenyatta University of Agriculture and Technology (JKUAT), Nairobi, Kenya

[3]Department of Mathematics, University of Yaounde 1, Yaounde, Cameroon

Abstract. In this paper, we consider some dividend problems in the perturbed compound Poisson risk model (Wiener Process) under constant barrier dividend strategy. We use the Fourier-sinc method to propose the estimators of the expected present dividend payments before ruin and the expected discounted penalty function also called Gerber-Shiu function using a random sample on claim number, individual claim sizes and surplus flow level. We show that our estimators have good convergence rate. We also derive some simulation examples to show the effectiveness of the estimators under finite sample. We simulate many graphs of ours estimators to show that show that our estimators are very closed and converge to the real curve.

Keywords: Fourier-sinc method; perturbed compound poisson risk model; surplus flow level; claim sizes; claim number; dividend payments; Gerber-Shiu function; Ruin; Poisson process; expected discounted penalty function.

2020 AMS Subject Classification: 62G05.

---

*Corresponding author

E-mail address: mrnoumegni@gmail.com

Received October 03, 2023

1. INTRODUCTION

Dividends strategy was introduced in actuarial research by Bruno De Finetti [1] in the binomial risk model. Since then, it has received attention of many scholars when studying dividend problems. See e.g. Zhang Liu and Ping Chen [2], Yongxia Zhao, Hua Dong, and Wei Zhong [3], Pecaric et al. [4] and Ronnie L Loeffe [5]. There are four performance measures that need to be known by insurance companies. The probability of ruin, the time of ruin, the surplus flows before and after ruin and the dividends shared up to ruin. The expected discounted penalty function also called Gerber-Shiu function is the joint distribution of the time to ruin, the surplus before ruin and the deficit at ruin. Clearly, the expected present value of dividend payments before ruin and the expected discounted penalty function are sufficient to evaluate the above mentioned performance measures. The the explicit formula for these two function are known under some specific claim size distributions such as Exponential, Erlang or Binomial, it remains problematic to obtain clear formula in general.

When analysing the expected present value of dividend or the expected discounted penalty function, insurance companies do not have the probability distribution of claim numbers and claims sizes, instead they have data set on claim numbers and individual claim sizes. As a result, they are unable to compute the explicit formulas, it is more useful to investigate non-parametric estimation of the aforementioned functions based on the data set of claim numbers and individual claim sizes. Estimation of ruin problems has been lately studied in risk theory in various models without dividend barriers. For instance, Honglong You et al. [6] proposed a bootstrap method to construct a confidence interval estimate of the ruin probability in the classical compound Poisson model. Yasutaka Shimizu and Zhimin Zhang [7] estimated the ruin probability under a spectrally negative Levy insurance risk by Laguerre serie expansion. Florian Dussap [8] and Wen Su, Yaodi Yong, and Zhimin Zhang [9] estimated the Geber-Shiu function under the perturbed compound Poisson risk model. Wen Su and Yunyun Wang [10] and Yujuan Huang et al. [11] considered a Levy risk model by Laguerre series expansion. Under the risk model with barrier dividend strategy, Jiayi Xie and Zhimin Zhang [12] estimated dividend problems in the classical compound Poisson risk model by Fourier cosine series expansion and Yang Yang, Jiayi Xie, and Zhimin Zhang [13] extended to the perturbed compound Poisson risk model. To the best of our knowladge, there are not any works on non-parametric estimation of dividend problems by Fourier-Sinc series expansion.

In this paper, we assume that the financial surplus U_t is modeled as Wiener process.

$$U_t = u + ct - \sum_{i=1}^{N_t} X_i + \sigma W(t), \qquad t \geq 0 \tag{1.1}$$

where $u \geq 0$ is the initial capital and $c > 0$ is the constant premium per time. The aggregate claims $\sum_{i=1}^{N_t} X_i$ follows a compound poisson process, where the number of claims $\{N_t\}_{t\geq 0}$ is an homogeneous poisson process with intensity $\lambda > 0$, and the individual claim sizes $\{X_i\}_{i\geq 1}$ is a sequence of positive i.i.d random variables generated by a generic variable X with density f_X and mean μ. Finally, $\{W(t)\}_{t\geq 0}$ is a standard Brownian motion with $W(0) = 0$, and $\sigma > 0$ is the diffusion volatility parameter. We suppose that $\{N_t\}$,$\{X_i\}$ and $\{W(t)\}$ are mutually independant. In this paper, we shall assume throughout the safety loading condition $c > \lambda\mu$, so that ruin is an uncertain event. Given a finite barrier level $b > 0$, we modify the process $\{U_t\}_t \geq 0$ by constant barrier dividend, and denote the modified model by $\left\{U_t^b\right\}_t \geq 0$. We assume that whenever the surplus process reaches level b, dividends are paid off continuously such that the surplus remains at level b until it falls bellow b. Let $\tau_b = \inf\left\{t \geq 0 : U_t^b \leq 0\right\}$ be the time of ruin. For the interested readers on ruin related problems, see F Lundberg and Förs*ä* akringsteknisk Riskutj*ä* amning [14], Corina Constantinescu et al. [15] and Filip Lundberg. 1. [16]. The present value of total dividends paid before ruin is given by

$$D_b = \int_b^{\tau_b} e^{-\delta t} dD(t), \qquad 0 \leq u \leq b$$

where $\delta > 0$ is the force of interest for valuation, and $D(t)$ is the aggregate dividends paid by time t. Given the initial surplus level $0 \leq u \leq b$, we are interested in the expected present value of total dividend payments before ruin $V(u,b)$ and the expected discounted penalty function $\phi(u,b)$ associated with model U_t^b and defined by

$$V(u,b) = \mathbb{E}\left[\int_0^{\tau_b} e^{-\delta t} dD_b(t) \mid U_0 = u\right] \tag{1.2}$$

and

$$\phi(u,b) = \mathbb{E}\left[e^{-\delta \tau_b} w\left(|U_{\tau_b}^b|\right) \mid U_0 = u\right] \tag{1.3}$$

w is a nonnegative penalty function of the deficit at ruin.

In this paper, we shall focus on the non-parametric estimation of the expected present value of dividend payments before ruin and the expected discounted penalty function. This problem has been considered by Yang Yang, Jiayi Xie, and Zhimin Zhang [13]. they proposed the estimators based on Fourier Cosine method. The main objective of this paper is to propose an alternative method to estimate $V(u,b)$ and $\phi(u,b)$, the Fourier-Sinc method. This method was used by Zhimin Zhang [17] to propose an estimator of the Gerber-Shiu function in a perturbed compound poisson risk model without dividend strategy. The remainder of this paper is orginized as follows. In section 2, we present some preliminaries on $V(u,b)$ and $\phi(u,b)$. In section 3, we describe the Fourier-Sinc method. In section 4, we study the estimators of $V(u,b)$ and $\phi(u,b)$ based on Fourier-Sinc series expansion. In section 5, we study the asymptotic properties of our estimators when the observation interval is large. Some simulation results are given in section 6 to show the effectiveness of our method and section 7 is the conclusion.

2. Some Preliminaries

In this section, we recall some important results that shall be used. We shall first define an explicite formula of $V(u,b)$. Note that the Wiener Process U_t is known to be a spectrally negative Levy process, and its Laplace exponent is defined by

$$\psi_U(s) = cs + \frac{1}{2}\sigma^2 s^2 - \lambda\left(1 - \mathscr{L}f_X(s)\right) \tag{2.1}$$

where $\mathscr{L}f_X(s) = \int_0^\infty e^{-sx} f_X(x)dx$ is the Laplace transform of f_X, see Masahiko Egami and Kazutoshi Yamazaki [18]. We define the q-scale function $W_q(x)$ associated with the process U_t, with Laplace transform given by

$$\int_0^\infty e^{-sx} W_q(x)dx = \frac{1}{\psi_U(s) - q}, \qquad for \quad s \geq \phi_q. \tag{2.2}$$

$W_q(x)$ is a strictly increasing and continuous function. One can extend W_q to the whole real line by setting $W_q(x) = 0$ for $x < 0$. Througout this work we shall consider the case $q = \delta$ and $\rho = \Phi_\delta$ for notational convenience. By Zhimin Zhang and Zhenyu Cui [19] we know that

$$h(x) = \frac{\delta}{\psi_U'(\rho)} e^{\rho x} - \delta W_\delta(x), \qquad x \in \mathbb{R} \tag{2.3}$$

is the probability density function of the random variable $U_0 - U_{e_\delta}$, where e_δ denotes an exponential random variable with rate $\delta \geq 0$. Set

$$h_+(x) = \begin{cases} h(x), & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$g_+(x) = \begin{cases} h'(x), & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The expected present value of dividend payments before ruin depends of h_+ and g_+ can be expressed by

$$V(u,b) = \frac{\frac{\delta}{\psi_U'(\rho)} - e^{\rho u} h_+(u)}{\frac{\delta \rho e^{\rho(b-u)}}{\psi_U'(\rho)} - e^{-\rho u} g_+(u)} \tag{2.4}$$

For futher details, see Jiayi Xie and Zhimin Zhang [12].

For $\{U_t\}_t \geq 0$, The ruin time is defined by $\tau = \inf\{t \geq 0 : U(t) < 0\}$ with the convention $\tau = \infty$ if $U(t) \geq 0$ for all $t \geq 0$ and according to this definition the expected discounted penalty function in this model is defined by

$$\phi(u) = \mathbb{E}\left[e^{-\delta\tau} w(U(\tau_-), |U(\tau)|) \, 1_{\{\tau<\infty\}} | \, U_0 = u\right], \quad u \geq 0 \tag{2.5}$$

where $w : [0,\infty) * [0,\infty) \mapsto [0,\infty)$ is a measurable penalty function of the surplus prior to ruin and the deficit at ruin, $\delta \geq 0$ represents the force of interest and $1_{\{A\}}$ is the indicator function of the event A. Set $w_0 = w(0,0)$. For the expected discounted penalty function $\phi(u,b)$, it can be given via the dividends-penalty identities

$$\phi(u,b) = \phi(u) + \phi'(b) V(u,b) \tag{2.6}$$

proposed by Yasutaka Shimizu and Zhimin Zhang [20].

3. Description of the Method

For all integrable function f such that the Fourier transform $\mathscr{F} f$ of f is absolutely integrable, one can recover the function f by the inverse transform formula

$$f(x) = \frac{1}{2\Pi} \int_{-\infty}^{+\infty} e^{-isx} \mathscr{F} f(s) ds \tag{3.1}$$

If f is not integrable, we can truncate the integral domain and use

$$f_m(x) = \frac{1}{2\Pi} \int_{-m\pi}^{+m\pi} e^{-isx} \mathscr{F} f(s) ds \tag{3.2}$$

to approximate the function f, where m is a large number. It is well known that $\|f - f_m\|^2 = O(m^{-1})$.

Let us introduce the following subset of $\mathbb{L}^2$

$$S_m = \left\{ f \in \mathbb{L}^2, Supp(\mathscr{F} f) \subset [-m\pi, m\pi] \right\}$$

where $Supp(\mathscr{F} f)$ denotes the support set of the Fourier transform $\mathscr{F} f$. Let $\sin(x) = \frac{\sin(\pi x)}{\pi x}$, and for $m > 0, \quad k \in \mathbb{Z}$, define

$$\psi_{m,k}(x) = \sqrt{m} sinc\,(mx - k)\,. \tag{3.3}$$

It is well known that $\left\{ \psi_{m,k} \right\}_{k \in \mathbb{Z}}$ form an orthogonal basis of S_m. Hence, for any $f \in S_m$, we have

$$f(x) = \sum \left\langle f, \psi_{m,k} \right\rangle \psi_{m,k}(x). \tag{3.4}$$

It follows from 3.4 that f_m has Fourier transform $\mathscr{F} f_m(s) = \mathscr{F} f(s)\, 1_{\{s \in [-m\pi, m\pi]\}}$, which together with plancherel theorem gives $\|f_m\|^2 = \frac{1}{2\pi} \|f_m\|^2 = \|f\|^2, \infty$. Hence, we conclude that $f_m \in S_m$. As a result, formula 3.4 yields

$$f_m(x) = \sum_{k \in \mathbb{Z}} A_{m,k} \psi_{m,k}(x) \tag{3.5}$$

where $A_{m,k} \left\langle f_m, \psi_{m,k} \right\rangle$. Note that $\psi_{m,k}$ has Fourier transform

$$\mathscr{F} \psi_{m,k}(s) = e^{\frac{isk/m}{\sqrt{m}}} 1_{\{s \in [-m\pi, m\pi]\}} \tag{3.6}$$

see Zhimin Zhang [17] which together with plancherel theorem gives $\left\| \psi_{m,k} \right\|_2^2 = \frac{1}{2\pi} \left\| \psi_{m,k} \right\|^2 = 1$. For the coefficient $A_{m,k}$, by the plancherel theorem, we have

$$
\begin{aligned}
A_{m,k} &= \frac{1}{2\pi}\left\langle \mathscr{F} f_m, \mathscr{F} \psi_{m,k}\right\rangle \\
&= \frac{1}{2\pi\sqrt{m}}\int_{-m\pi}^{m\pi} \mathscr{F} f(s) e^{-isk/m} ds \\
&= \sqrt{m}\int_{\frac{-1}{2}}^{\frac{1}{2}} \mathscr{F} f\left(2\pi ms\right) e^{-i2\pi sk} ds \\
&= \sqrt{m}\int_{0}^{1} \mathscr{F} f\left(2\pi m\left(s-\frac{1}{2}\right)\right) e^{-i2\pi\left(s-\frac{1}{2}\right)k} ds \\
&= \sqrt{m}e^{ik\pi}\int_{0}^{1} \mathscr{F} f\left(2\pi m\left(s-\frac{1}{2}\right)\right) e^{-i2\pi sk} ds
\end{aligned} \tag{3.7}
$$

Moreover, for every fixed m, plancherel theorem gives

$$
\sum_{k\in\mathbb{Z}} \left|A_{m,k}\right|^2 = \left\|\sum_{k\in\mathbb{Z}} A_{m,k}\psi_{m,k}\right\|^2 = \|f_m\|^2 = \frac{1}{2\pi}\|\mathscr{F} f_m\|^2 \le \frac{1}{2\pi}\|\mathscr{F} f\|^2 = \|f\|^2 < \infty
$$

which implies that $\left|A_{m,k}\right| \to 0$ as $|k| \to \infty$. As a result, we can truncate the infinite sum in 3.5 to get

$$
f_m(x) = f_{m,k}(x) := \sum_{k=-K}^{K} A_{m,k}\psi_{m,k}(x) \tag{3.8}
$$

where K is a large integer.

4. ESTIMATION PROCEDURE

In this section, we are going to estimate the expected future dividends and the expected penalty function using the Fourier-Sinc series expansion.We assume that the surplus process can be observed over a long time interval $[0,T]$. Let $\Delta \ge 0$ be a sampling interval. Without loss of generality, we assume that T/Δ is an integer and let $n = T/\Delta$.

Suppose that the insurer can get the following data-set.

- Data-set on surplus level: $\left\{U_{j\Delta} : j = 0,1,2,\cdots,n\right\}$ Where $U_{j\Delta}$ is the observe surplus level at time $t = j\Delta$
- Data-set on claim numbers and claim sizes:
  $\left\{N_{j\Delta}, X_1, X_2, \cdots, X_{N_{j\Delta}}\right\}$ $\quad j = 0,1,2,\cdots,N_T$ where $N_{j\Delta}$ is the total claim number up to time $t = j\Delta$.

We shall propose an estimator for the expected discounted penalty function $V(u,b)$ and $\phi(u,b)$. Obviously, we need to estimate the following quantities $\sigma^2,\ \lambda,\ \rho,\ \psi_U'(\rho),\ \mathscr{F}f_X(s),\ h_+(u)$ and $g_+(u)$. As in Zhimin Zhang [17], we can estimate $\lambda,\ \rho,\ \mathscr{F}f_X(s)$ and $\mathscr{L}f_X(s)$ by

$$\widehat{\sigma}^2 = \frac{1}{n\Delta}\sum_{j=1}^{N_T}\left[U_{j\Delta} - U_{(j-1)\Delta} - c\Delta + \sum_{k=N_{(j-1)\Delta+1}}^{N_{j\Delta}} X_k\right]^2$$

$$\widehat{\lambda} = \frac{N_T}{T}$$

$$\widehat{\mathscr{F}f}_X(s) = \frac{1}{N_T}\sum_{j=1}^{N_T} e^{isX_j}$$

$$\widehat{\mathscr{L}f}_X(s) = \frac{1}{N_T}\sum_{j=1}^{N_T} e^{-sX_j}$$

It is known that $\widehat{\rho} \in \left[\left(-c+\sqrt{c^2+2\widehat{\sigma}^2\delta}\right)/\widehat{\sigma}^2 \quad , \quad \frac{\left(\widehat{\lambda}+\delta\right)}{c}\right]$ and $\widehat{\rho} \geq \delta/c$. So it remains to estimate $h_+(u)$ and $g_+(u)$ which give the estimation of the expected present value of total dividends $V(u,b)$ by formula 2.4. Then we shall estimate the first derivative $\phi'(u)$ of the expected penalty function and finally, we use the penalty identity function 2.6 to have the estimators of the expected penalty function $\phi(u,b)$.

First, h_+ and g_+. Since we have their Fourier transforms, we need to estimatte the coefficients $\left\{A_{m,k}^{h_+}\right\}$ and $\left\{A_{m,k}^{g_+}\right\}$ by formula 3.7 and then we get the estimates of h_+ and g_+ based on formula 3.8. We recall that the Fourier transforms of h_+ and g_+ are given by

$$\mathscr{F}h_+(s) = \frac{\delta}{\psi_U'(\rho)} \times \frac{1}{-is-\rho} - \frac{\delta}{\psi_U(-is)-\delta}$$

and

$$\mathscr{F}g_+(s) = \frac{\delta}{\psi_U'(\rho)} \times \frac{\rho}{-is-\rho} - \frac{-i\delta s}{\psi_U(-is)-\delta}$$

It follows from formula 3.8 that h_+ and g_+ can be estimated by

$$\widehat{h_{+}}_{m,k} = \sum_{k=-K}^{K} \widehat{A}_{m,k}^{h_+}\psi_{m,k}(u) \tag{4.1}$$

and

$$\widehat{g_{+}}_{m,k} = \sum_{k=-K}^{K} \widehat{A}_{m,k}^{g_+}\psi_{m,k}(u) \tag{4.2}$$

where

$$\widehat{A}^{h_+}_{m,k} = \frac{1}{2\pi\sqrt{m}}\int_{-m\pi}^{m\pi}\widehat{\mathscr{F}h}_+(s)e^{\frac{-isk}{m}}ds \tag{4.3}$$

and

$$\widehat{A}^{g_+}_{m,k} = \frac{1}{2\pi\sqrt{m}}\int_{-m\pi}^{m\pi}\widehat{\mathscr{F}g}_+(s)e^{\frac{-isk}{m}}ds. \tag{4.4}$$

Now we can estimate the expected present value of total dividends by

$$\widehat{V}(u,b) = \frac{\frac{\delta}{\widehat{\psi_U}'(\widehat{\rho})} - e^{-\widehat{\rho}u}\widehat{h}_+(u)}{\frac{\delta\widehat{\rho}e^{\widehat{\rho}(b-u)}}{\widehat{\psi_U}'(\widehat{\rho})} - e^{-\widehat{\rho}u}\widehat{g}_+(u)} \tag{4.5}$$

Next, we use the Fourier-Sinc serie expqnsion to estimate our expected penalty function $\phi(u,b)$. Zhimin Zhang [17] already estimated the expected penalty function $\phi(u)$ in the perturbed compound poisson risk model with zero dividend. Hence, it remains to estimate its first derivative and then apply the dividends-penalty identities 2.6. We recall that

$$\mathscr{F}\phi'(s) = -is\mathscr{F}\phi(s) - \phi(0) \tag{4.6}$$

where

$$\mathscr{F}\phi(s) = \frac{\frac{\sigma^2}{2}w_0(-is-\rho) + \lambda\left[\mathscr{L}\omega(\rho) - \mathscr{F}\omega(s)\right]}{\psi_U(-is)-\delta}$$

$w_0 = w(0,0)$ and $\omega(u) = \int_u^\infty w(x-u)f_X(x)dx$.
By formula 3.8, ϕ' can be estimated by

$$\widehat{\phi'}_{m,k}(u) := \sum_{k=-K}^{K}\widehat{A}^{\phi'}_{m,k}\psi_{m,k}(u) \tag{4.7}$$

where

$$\widehat{A}^{\phi'}_{m,k} = \frac{1}{2\pi\sqrt{m}}\int_{-m\pi}^{m\pi}\widehat{F\phi'}(s)e^{\frac{-isk}{m}}ds$$

Finally, by the dividends-penalty identities 2.6 we obtain

$$\widehat{\phi}(u,b) = \widehat{\phi}(u) + \widehat{\phi}'(b)V(u,b)$$

Remark 4.1. *When coming from the point of view of computation, it is more convenient to express 3.8 as follows*

$$f_{m,k}(x) := \sum_{k=-K}^{K} \widehat{B}_{m,k}\psi_{m,k}(x)$$

where 3.7 and Trapezoidal rule,

$$\begin{aligned}\widehat{B}_{m,k} &= \widehat{A}_{m,k-K-1}\\ &= \sqrt{m}e^{\pi(k-K-1)}\int_0^1 \mathscr{F}f\left(2\pi m\left(s-\frac{1}{2}\right)\right)e^{-i2\pi sK}e^{-i2\pi s(k-1)}ds\\ &\approx \sqrt{m}e^{\pi(k-K-1)}\sum_{j=1}^{2K+1}\frac{1}{2K+1}\mathscr{F}f\left(2\pi m\left(\frac{j-1}{2K+1}-\frac{1}{2}\right)\right)e^{-i2\pi s\frac{k}{2K+1}(j-1)}e^{-i\frac{2\pi}{2K+1}(j-1)(k-1)}\end{aligned}$$

Hence, the coefficient $\widehat{B}_{m,k}$ can be computed via Fast fourier transform, FFT.

5. Asymptotic Properties

In this section, we derive some asymptotic properties of the estimators as $T \to \infty$. Throughout this section, we use C to denote a generic constant that may vary at different steps. For two nonnegative functions f_1, f_2 with domain $\chi \subseteq \mathbb{R}$, we use $f_1 \leq f_2$ to mean $f_1(x) \leq Cf_2(x)$ uniformly in $x \in \chi$. Similarly, we use $f_1 \geq f_2$ to mean $f_1(x) \geq Cf_2(x)$ uniformly in $x \in \chi$. For two sequences of functions $\{f_k\}$ and $\{g_k\}$, we use $f_k \leq (or \geq) g_k$ to mean $f_k(x) \leq (or \geq) Cg_k(x)$ uniformly in k and x.

Proposition 5.1. *Suppose that $c \geq \lambda\mu_1$ and $\mu_2 \leq \infty$ hold, the we have $\widehat{\rho} - \rho = O_p\left(T^{-1/2}\right)$.*

Proposition 5.2. *Suppose that $C > \lambda\mu_1$, $\mu_2 < \infty$, $\left\|H_j(X)\right\|_{p,1} < \infty$ for $j = 0, 1$, the we have*

$$|\mathscr{F}h_+(s)| \lesssim \frac{1}{1\vee|s|} \qquad |\mathscr{F}g_+(s)| \lesssim \frac{1}{1\vee|s|}$$

$$\left|\frac{d}{ds}\mathscr{F}h_+(s)\right| \lesssim \frac{1}{1\vee s^2} \qquad \left|\frac{d}{ds}\mathscr{F}g_+(s)\right| \lesssim \frac{1}{1\vee s^2}$$

Proof see Proposition 2 in Zhimin Zhang [17].

Proposition 5.3. *Suppose that $C > \lambda\mu_1$, $\mu_2 < \infty$, $\left\|H_j(X)\right\|_{p,1} < \infty$ for $j = 0, 1$ and $\left\|H_j(X)\right\|_{p,2} < \infty$ for $j = 1, 2$. Then for large m, K, T, we have*

$$\sup_{s\in[-m\pi,m\pi]}\left|\widehat{\mathscr{F}h_+}(s)-\mathscr{F}h_+(s)\right|=O_p\left(\sqrt{(\log m)/T}\right) \tag{5.1}$$

$$\sup_{s\in[-m\pi,m\pi]}\left|\widehat{\mathscr{F}g_+}(s)-\mathscr{F}g_+(s)\right|=O_p\left(\sqrt{(\log m)/T}\right) \tag{5.2}$$

Proof

First, we have

$$\widehat{\mathscr{F}h_+}(s)-\mathscr{F}h_+(s)=\left(\frac{\delta}{\widehat{\psi}'_U(\widehat{\rho})}.\frac{1}{-is-\widehat{\rho}}-\frac{\delta}{\psi'_U(\rho)}.\frac{1}{-is-\rho}\right)+\left(\frac{\delta}{\psi_U(-is)-\delta}-\frac{\delta}{\widehat{\psi}_U(-is)-\delta}\right) \tag{5.3}$$

$$=I_h(s)+II_h(s) \tag{5.4}$$

$$\begin{aligned}
I_h(s)&\leq\left|\frac{\delta}{\widehat{\psi}'_U(\widehat{\rho})}\left(\frac{1}{-is-\widehat{\rho}}-\frac{1}{-is-\rho}\right)\right|+\left|\left(\frac{\delta}{\widehat{\psi}'_U(\widehat{\rho})}-\frac{\delta}{\psi'_U(\rho)}\right)\frac{1}{-is-\rho}\right|\\
&=\frac{\delta}{\left|\widehat{\psi}'_U(\widehat{\rho})\right|}.\frac{|\widehat{\rho}-\rho|}{|-is-\widehat{\rho}|.|-is-\rho|}+\frac{\delta}{\left|\widehat{\psi}'_U(\widehat{\rho})\right|\left|\psi'_U(\rho)\right|}.\frac{|\widehat{\psi}'_U(\widehat{\rho})-\psi'_U(\rho)|}{|-is-\rho|}\\
&\leq\frac{\delta}{\left|\widehat{\psi}'_U(\widehat{\rho})\right|}\frac{|\widehat{\rho}-\rho|}{\widehat{\rho}\rho}+\frac{\delta}{\left|\widehat{\psi}'_U(\widehat{\rho})\right|.\left|\psi'_U(\rho)\right|}\frac{|\widehat{\psi}'_U(\widehat{\rho})-\psi'_U(\rho)|}{\rho}
\end{aligned}$$

As $\widehat{\rho}-\rho=O_p\left(T^{-1/2}\right)$ and $\widehat{\psi}'_U(\widehat{\rho})-\psi'_U(\rho)=O_p\left(T^{-1/2}\right)$, it follows that

$$\sup_{s\in[-m\pi,m\pi]}|I_h(s)|=O_p(T^{-1/2}) \tag{5.5}$$

Let us consider $II_h(s)$, note that ρ is the positive root of equation $\psi_U(s)=\delta$. Then we have

$$\begin{aligned}
\psi_U(-is)-\delta&=[\psi_U(-is)-\delta]-[\psi_U(\rho)-\delta]\\
&=-ics-c\rho+\lambda\left[\mathscr{F}f_X(s)-\mathscr{L}f_X(s)\right]\\
&=(is+\rho)\left\{-c+\lambda\int_0^\infty\frac{e^{isx}-e^{-\rho x}}{is+\rho}f_X(x)dx\right\}\\
&=(is+\rho)\left\{-c+\lambda\int_0^\infty e^{isx}\int_0^x e^{-(\rho+is)y}dyf_X(x)dx\right\}
\end{aligned}$$

which yields that for real number s,

$$|\psi_U(-is)-\delta|\geq|is+\rho|.\left\{c-\lambda\left|\int_0^\infty e^{isx}\int_0^x e^{-(\rho+is)y}dyf_X(x)dx\right|\right\}$$

$$\geq \rho\left(c-\lambda\int_0^\infty xf_X(x)dx\right)=\rho\left(c-\lambda E(X)\right)$$

On the other hand, we have

$$\begin{aligned}\widehat{\psi}_U(-is)-\psi_U(-is) &= (\widehat{\lambda}-\lambda)-\left(\frac{1}{T}\sum_{j=1}^{N_T}e^{isX_j}-\lambda E(e^{isX})\right)\\ &= (\widehat{\lambda}-\lambda)\left(1-E(e^{isX})\right)-\widehat{\lambda}\left(\frac{1}{N_T}\sum_{j=1}^{N_T}e^{isX_j}-E(e^{isX})\right)\end{aligned}$$

We then obtain

$$\sup_{s\in[-m\pi,m\pi]}|\widehat{\psi}_U(-is)-\psi_U(-is)|\leq 2\left|\widehat{\lambda}-\lambda\right|+\widehat{\lambda}\sup_{s\in[-m\pi,m\pi]}\left|\frac{1}{N_T}\sum_{j=1}^{N_T}e^{isX_j}-E(e^{isX})\right|$$

It is known that $\widehat{\lambda}-\lambda=O_p(T^{-1/2})$.

It remains to study the uniform convergence of

$$\frac{1}{T}\sum_{j=1}^{N_T}e^{isX_j}-E(e^{isX}).$$

$$\frac{1}{T}\sum_{j=1}^{N_T}e^{isX_j}-E(e^{isX})=\frac{1}{T}\sum_{j=1}^{N_T}\left[g_S(X_j)-E\left(g_s(X_j)\right)\right]+\left(\frac{N_T}{T}-\lambda\right)E\left(g_s(X)\right)$$

where $g_s(x)=e^{isx}$.

One can see in Zhimin Zhang [17] that

$$\sup_{s\in[-m\pi,m\pi]}\left|\left(\frac{N_T}{T}-\lambda\right)E\left(g_s(X)\right)\right|=O_p(T^{-1/2})$$

For the uniform convergence of $\frac{1}{T}\sum_{j=1}^{N_T}\left[g_S(X_j)-E\left(g_s(X_j)\right)\right]$, we introduce two classes of real-values functions

$$\mathscr{G}_{m,R}=\{g:g=\mathrm{Re}\ (g_s)\quad for\quad some\quad s\in[-m\pi,m\pi]\}$$

$$\mathscr{G}_{m,I}=\{g:g=\mathrm{Im}\ (g_s)\quad for\quad some\quad s\in[-m\pi,m\pi]\}$$

where Re $(.)$ and Im $(.)$ denote the real part and the imaginary part respectively.

Then,

$$\sup_{s\in[-m\pi,m\pi]}\left|\frac{1}{T}\sum_{j=1}^{N_T}\left[g_S(X_j)-E\left(g_s(X_j)\right)\right]\right|$$
$$\leq \sup_{g\in\mathscr{G}_{m,R}}\left|\frac{1}{T}\sum_{j=1}^{N_T}\left[g(X_j)-E\left(g(X_j)\right)\right]\right|+\sup_{g\in\mathscr{G}_{m,I}}\left|\frac{1}{T}\sum_{j=1}^{N_T}\left[g(X_j)-E\left(g(X_j)\right)\right]\right|$$

We shall only study the convergence rate of $\sup_{g\in\mathscr{G}_{m,R}}\left|\frac{1}{T}\sum_{j=1}^{N_T}\left[g(X_j)-E\left(g(X_j)\right)\right]\right|$ since the other one is similar.

From Zhimin Zhang [17], it can be seen that for every $\delta>0$, the bracketing integral $\mathrm{J}_{[]}\left(\delta,\mathscr{G}_{m,R},\mathbb{L}^2\right)\leq\sqrt{\log m}$ which immediatly gives with the corollary **19**.**35** in Aad W Van der Vaart [21], the following

$$E\left(\sup_{g\in\mathscr{G}_{m,R}}\left|\frac{1}{T}\sum_{j=1}^{N_T}\left[g(X_j)-E\left(g(X_j)\right)\right]\right|\right)$$
$$E\left(\frac{\sqrt{N_T}}{T}E\left(\frac{1}{\sqrt{N_T}}\sup_{g\in\mathscr{G}_{m,R}}\left|\sum_{j=1}^{N_T}\left[g(X_j)-E\left(g(X_j)\right)\right]\right|/N_T\right)\right)$$
$$\leq\frac{\sqrt{\log m}}{T}E(\sqrt{N_T})$$
$$\leq\frac{\sqrt{\log m}}{T}E(N_T)=\sqrt{\frac{\lambda}{T}\log m}$$

which implies that

$$\sup_{g\in\mathscr{G}_{m,R}}\left|\frac{1}{T}\sum_{j=1}^{N_T}\left[g(X_j)-E\left(g(X_j)\right)\right]\right|=O_p\left(\sqrt{\log m/T}\right)\tag{5.6}$$

Similarly, we show that

$$\sup_{g\in\mathscr{G}_{m,I}}\left|\frac{1}{T}\sum_{j=1}^{N_T}\left[g(X_j)-E\left(g(X_j)\right)\right]\right|=O_p\left(\sqrt{\log m/T}\right)\tag{5.7}$$

Then by 5.6 and 5.7, we have

$$\sup_{s\in[-m\pi,m\pi]}|II_h(s)|=O_p(\sqrt{\log m/T})\tag{5.8}$$

Hence 5.3, 5.5 and 5.8 give the end of the proof.

Similarly, we show the uniform convergence of $\widehat{\mathscr{F}g_+}(s)$.

Proposition 5.4. *Suppose that $c > \lambda\mu_1$, $\mu_2 < \infty$, $\left\|H_j(X)\right\|_{p,1} < \infty$ for $j = 0,1$ and $\left\|H_j(X)\right\|_{p,2} < \infty$ for $j = 1,2$. Then for large m,K,T, we have*

$$\sup_{s\in[-m\pi,m\pi]}\left|\frac{d}{ds}\widehat{\mathscr{F}h_+}(s) - \frac{d}{ds}\mathscr{F}h_+(s)\right| = O_p\left(\sqrt{(\log m)/T}\right) \tag{5.9}$$

$$\sup_{s\in[-m\pi,m\pi]}\left|\frac{d}{ds}\widehat{\mathscr{F}g_+}(s) - \frac{d}{ds}\mathscr{F}g_+(s)\right| = O_p\left(\sqrt{(\log m)/T}\right) \tag{5.10}$$

Proof The proof of this proposition is similar to the previous proposition

Theorem 5.1. *Suppose that $c > \lambda\mu_1$, $\mu_2 < \infty$, $\left\|H_j(X)\right\|_{p,1} < \infty$ for $j = 0,1$ and $\left\|H_j(X)\right\|_{p,2} < \infty$ for $j = 1,2$. Then for large m,K,T, we have*

$$\left\|\widehat{h_{+}}_{m,k} - h_+\right\|^2 = O(m^{-1}) + O(m/K) + O_p\left(\sqrt{m(\log m)/T}\right)$$

$$\left\|\widehat{g_{+}}_{m,k} - g_+\right\|^2 = O(m^{-1}) + O(m/K) + O_p\left(\sqrt{m(\log m)/T}\right)$$

Proof For $\left\|\widehat{h_{+}}_{m,k} - h_+\right\|$, by triangle inequality, we have

$$\begin{aligned}\left\|\widehat{h_{+}}_{m,k} - h_+\right\| &= \left\|\widehat{h_{+}}_{m,k} - h_{+m,k} + h_{+m,k} + h_{+m} - h_{+m} - h_+\right\| \\ &\le \left\|\widehat{h_{+}}_{m,k} - h_{+m,k}\right\| + \left\|h_{+m,k} - h_{+m}\right\| + \left\|h_{+m} - h_+\right\|\end{aligned} \tag{5.11}$$

By Plancherel theorem, we have

$$\|h_{+m} - h_+\|^2 \le \frac{1}{2\pi}\|\mathscr{F}h_{+m} - \mathscr{F}h_+\|^2 = \frac{1}{2\pi}\int_{|s||>m\pi}|\mathscr{F}h_+(s)|^2 ds \lesssim \frac{1}{2\pi}\int_{|s||>m\pi}\frac{1}{s^2}ds = O(m^{-1}) \tag{5.12}$$

Secondly, it is known that $\|\psi_k\| = 1$ and $\langle\psi_{m,k},\psi_{m,j}\rangle = 0$ for $k \neq j$. So we have

$$\left\|\mathscr{F}h_{+m,k} - \mathscr{F}h_{+m}\right\|^2 = \left\|\sum_{|k|>K}A^{h_+}_{m,k}\psi_{m,k}\right\|^2 = \sum_{|k|>K}\left|A^{h_+}_{m,k}\right|^2 \tag{5.13}$$

$$\begin{aligned}A^{h_+}_{m,k} &= \frac{1}{2\pi\sqrt{m}}\int_{-m\pi}^{m\pi}\mathscr{F}h_+(s)ds \\ &= \frac{1}{2\pi\sqrt{m}}\frac{1}{\frac{-ik}{m}}\int_{-m\pi}^{m\pi}\mathscr{F}h_+(s)de^{\frac{-isk}{m}} \\ &= \frac{i\sqrt{m}}{2\pi k}\left(\mathscr{F}h_+(m\pi)e^{-ik\pi} - \mathscr{F}h_+(-m\pi)e^{ik\pi}\right) - \int_{-m\pi}^{m\pi}e^{-isk/m}\frac{d}{ds}\mathscr{F}h_+(s)ds\end{aligned} \tag{5.14}$$

Since $\mathscr{F}h_+$ is a bounded function and by the third enequation in proposition 5.2, we have

$$\left|\int_{-m\pi}^{m\pi} e^{-isk/m}\frac{d}{ds}\mathscr{F}h_+(s)ds\right| \leq \int_{-\infty}^{\infty}\left|\frac{d}{ds}\mathscr{F}h+(s)\right|ds < \infty$$

Hence formula 5.14 gives $\left|A_{m,k}\right| \lesssim \frac{\sqrt{m}}{k}$

Then formula 5.13 gives

$$\left\|h_{+m,k} - h_{+m}\right\|^2 \lesssim \sum_{|k|>K}\frac{m}{k^2} = O(m/K) \tag{5.15}$$

Finally, let us consider the quantity $\left\|\widehat{h}_{+m,k} - h_{m,k}\right\|$,

$$\begin{aligned}\left\|\widehat{h}_{+m,k} - h_{m,k}\right\|^2 &= \left\|\sum_{k=-K}^{K}\left(\widehat{A}_{m,k}^{h_+} - A_{m,k}^{h_+}\right)\psi_{m,k}\right\|^2 \\ &= \sum_{k=-K}^{K}\left|\widehat{A}_{m,k}^{h_+} - A_{m,k}^{h_+}\right|^2 \end{aligned}\tag{5.16}$$

By formula 5.14, we have

$$\begin{aligned}\widehat{A}_{m,k}^{h_+} - A_{m,k}^{h_+} = \frac{i\sqrt{m}}{2\pi k}&\left(\left[\widehat{\mathscr{F}h}_+(m\pi) - \mathscr{F}h_+(m\pi)\right]e^{-ik\pi} - \left[\widehat{\mathscr{F}h}_+(-m\pi) - \mathscr{F}h_+(-m\pi)\right]e^{ik\pi}\right) \\ -\frac{i\sqrt{m}}{2\pi k}&\int_{-m\pi}^{m\pi} e^{-isk/m}\left(\frac{d}{ds}\widehat{\mathscr{F}h}_+(s)ds - \frac{d}{ds}\mathscr{F}h_+(s)ds\right)\end{aligned}\tag{5.17}$$

Then formula 5.17 and proposition 5.3 and 5.4 gives

$$\begin{aligned}\left|\widehat{A}_{m,k}^{h_+} - A_{m,k}^{h_+}\right| &\leq \frac{\sqrt{m}}{2\pi k}\left(\left|\widehat{\mathscr{F}h}_+(m\pi) - \mathscr{F}h_+(m\pi)\right| + \left|\widehat{\mathscr{F}h}_+(-m\pi) - \mathscr{F}h_+(-m\pi)\right|\right) \\ &+ \frac{\sqrt{m}}{2\pi k}\int_{-m\pi}^{m\pi}\left|\frac{d}{ds}\widehat{\mathscr{F}h}_+(s)ds - \frac{d}{ds}\mathscr{F}h_+(s)ds\right| \\ &\leq \frac{1}{K}O_p\left(\sqrt{m\log m/T}\right)\end{aligned}\tag{5.18}$$

Then formula 5.16 yields

$$\left\|\widehat{h}_{+m,k} - h_{m,k}\right\|^2 = O_p\left(\sqrt{m\log m/T}\right) \tag{5.19}$$

Finally, by 5.11, 5.12, 5.15 and 5.19, we obtain

$$\left\|\widehat{h_{+m,k}} - h_+\right\| = O(m^{-1}) + O(m/K) + O_p\left(\sqrt{m(\log m)/T}\right)$$

Similary, we show the proof of the second equality of the proof.

Proposition 5.5. *Suppose that* $c > \lambda\mu_1$, $\mu_2 < \infty$, $\left\|H_j(X)\right\|_{p,1} < \infty$ *for* $j = 0,1$ *and* $\left\|H_j(X)\right\|_{p,2} < \infty$ *for* $j = 1,2$. *Then for large* m, K, T, *we have*

$$\left\|\widehat{V}(u,b) - V(u,b)\right\|^2 = O(m^{-1}) + O(m/K) + O_p\left(\sqrt{m(\log m)/T}\right)$$

Proposition 5.6. *Suppose that* $c > \lambda\mu_1$, $\mu_2 < \infty$, $\left\|H_j(X)\right\|_{p,1} < \infty$ *for* $j = 0,1$ *and* $\left\|H_j(X)\right\|_{p,2} < \infty$ *for* $j = 1,2$. *Then for large* m, K, T, *we have*

$$\left\|\widehat{\phi}(u,b) - \phi(u,b)\right\|^2 = O(m^{-1}) + O(m/K) + O_p\left(\sqrt{m(\log m)/T}\right)$$

6. Numerical Simulation

In this section, we provide some numerical results to show that our method is effective. All computations are done in MATLAB on a EliteBook, with Intel(R) Core(TM) i5-6300U CPU@2.40GHz 2.50GHz and a RAM of 8GB. Throughout this section, we set $c = 8$, $\lambda = 5$, $\delta = 0.1, \sigma = 1$ and use claim size density functions, the exponential EXP(1): $f_X(x) = e^{-x}, \quad x > 0$ and the Erlang(2,2): $f_X(x) = 4xe^{-2x}, \quad x > 0$. Then for these parameters, the explicit formula of the expected present value of dividend payments can be easily computed by formula 2.4 and the explicit formula of the Gerber-Shiu function in the Wiener Process without dividend strategy can be found in Zhimin Zhang [17]. Now for these two claim size density functions, closed form of Fourier transforms exist so that the Fourier -Sinc method estmation of $h_+(u)$ and $g_+(u)$ can be computed. Regarding the observation period $[0,T]$, we shall take $T = 1000 * p$ for $p = 1,2,3,4,5$. Formula 4.1 and 4.2 are used to compute the estimators $\widehat{h_{+m,k}}$ and $\widehat{g_{+m,k}}$ where the coefficients $\widehat{A}^{h_+}_{m,k}$ and $\widehat{A}^{g_+}_{m,k}$ are computed via FFT algorithm as suggested in Remark4.1. We take $m = 20$, $K = 2^{10}$ and we repeat 300 simulations.

First, we compute the empirical average relative errors and the average absolute errors for $\widehat{V}(u,b)$ and $\widehat{\phi}(u,b)$. Which are defined by

$$Average \quad relative \quad errors: \quad \frac{1}{\sharp\mathscr{U}} \sum_{u\in\mathscr{U}} \frac{1}{300} \sum_{j=1}^{300} \frac{\left|\widehat{V}_j(u,b) - V(u,b)\right|}{V(u,b)}$$

$$\frac{1}{\sharp\mathscr{U}'} \sum_{u\in\mathscr{U}'} \frac{1}{300} \sum_{j=1}^{300} \frac{\left|\widehat{\phi}_j(u,b) - \phi(u,b)\right|}{\phi(u,b)}$$

$$\textit{Average absolute errors}: \quad \frac{1}{\sharp\mathscr{U}}\sum_{u\in\mathscr{U}}\frac{1}{300}\sum_{j=1}^{300}\left|\widehat{V}_j(u,b)-V(u,b)\right|$$

$$\frac{1}{\sharp\mathscr{U}'}\sum_{u\in\mathscr{U}}\frac{1}{300}\sum_{j=1}^{300}\left|\widehat{\phi}_j(u,b)-\phi(u,b)\right|$$

where $\widehat{V}_j(u,b)$ and $\widehat{\phi}_j(u,b)$ denote the j^{th} simulation values of $\widehat{V}(u,b)$ and $\widehat{\phi}(u,b)$ respectively. $\mathscr{U}=\{1,2,\cdots,30\}$, $\mathscr{U}'=\{1,2,\cdots,15\}$. Since for $u>15$, $\widehat{\phi}(u,b)$ becomes too small. The empirical estimation errors for $\widehat{V}(u,b)$ are presented in **table** 1. we observe that both the empirical average errors and empirical absolute errors are decreasing w.r.t p which is due to that, as T increases, more sample is used to estmate $\widehat{V}(u,b)$ and the estimate values are better. **Table** 2 shows similar results when estimating the Gerber-Shiu function. We also observe that the empirical average relative errors are smaller than the empirical average absolute errors for $\widehat{V}(u,b)$, but the empirical average relative errors are larger than the empirical average absolute errors for $\widehat{\phi}(u,b)$. these observations can be explained by the fact that dividends are greater than one whereas the expected discounted penalty function is less than one.

Finally, we plot 300 consecutive estimators(green curves) on the same picture together with the true curve(red curves) under the exponential distribution to illustrate variability bands and demonstrate the stability of the method. We can see that estimator's beams are extremely close to true curve. In particular, it follows that for a large observation period, the variances of the estimation are very small. For our models, it is obvious that all of the lines increse with u for the the expected present value of total dividends wchich is consistent with the real world where the expected present value of total dividend payments before ruin increases with the initial surplus. Besides, we can observe on **figure**1 as T increases, the estimator $\widehat{V}(u,b)$ tends to be stable and converges to $V(u,b)$. **Figure**2 also investigates the function $\phi(u,b)$. We find that $\phi(u,b)$ is a decreasing function of the initial surplus u, which implies that when u is small, there is a higher chance for the ruin to occur. Although we only illustrate the cases with exponential and Erlang(2,2) claim size densities, The same behevior will be observed when using the other densities.

Empirical average relative errors			Empirical average absolute errors	
T	Exp(1)	Erlang(2,2)	Exp(1)	Erlang(2,2)
1000	0.08584	0.08238	0.38767	0.35876
2000	0.08346	0.07745	0.34789	0.30919
3000	0.07824	0.07186	0.28476	0.26845
4000	0.5782	0.05068	0.22089	0.21369
5000	0.4683	0.03894	0.20376	0.19286

TABLE 1. Estimation errors of $V(u,b)$ by Fourier-Sinc Method

Empirical average relative errors			Empirical average absolute errors	
T	Exp(1)	Erlang(2,2)	Exp(1)	Erlang(2,2)
1000	0.9479	0.9238	0.07893	0.07264
2000	0.8942	0.8662	0.07258	0.06329
3000	0.8527	0.8284	0.06530	0.05246
4000	0.6996	0.5839	0.05932	0.03596
5000	0.4365	0.3860	0.02953	0.01852

TABLE 2. Estimation errors of $\phi(u,b)$ by Fourier-Sinc Method

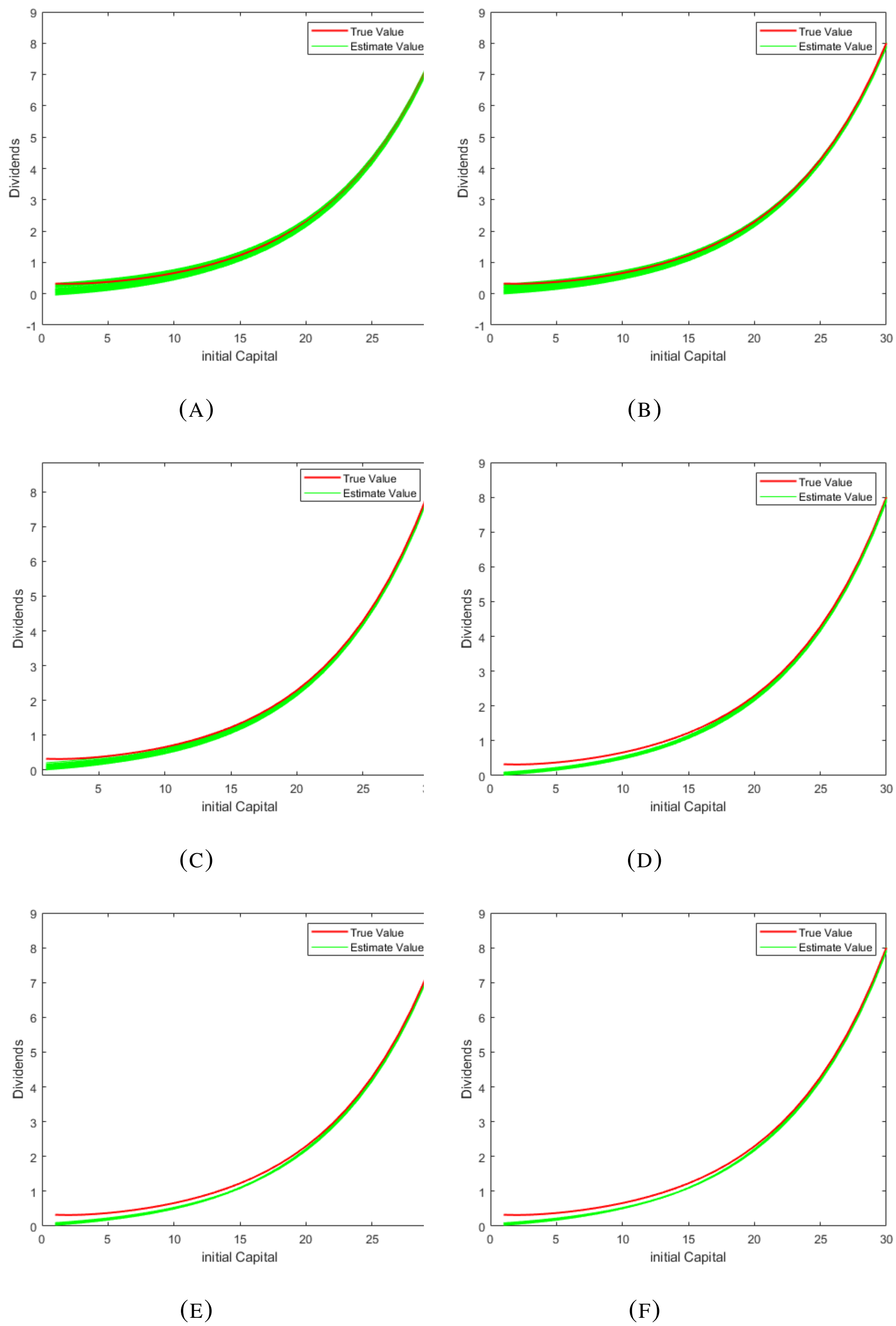

FIGURE 1. Beams for estimating the $V(u,b)$: 300 estimators in green, and the true value in bold red. (a) q=1000; (b) q=2000; (c) q=3000; (d) q=4000; (e) q=5000; (f) q=6000

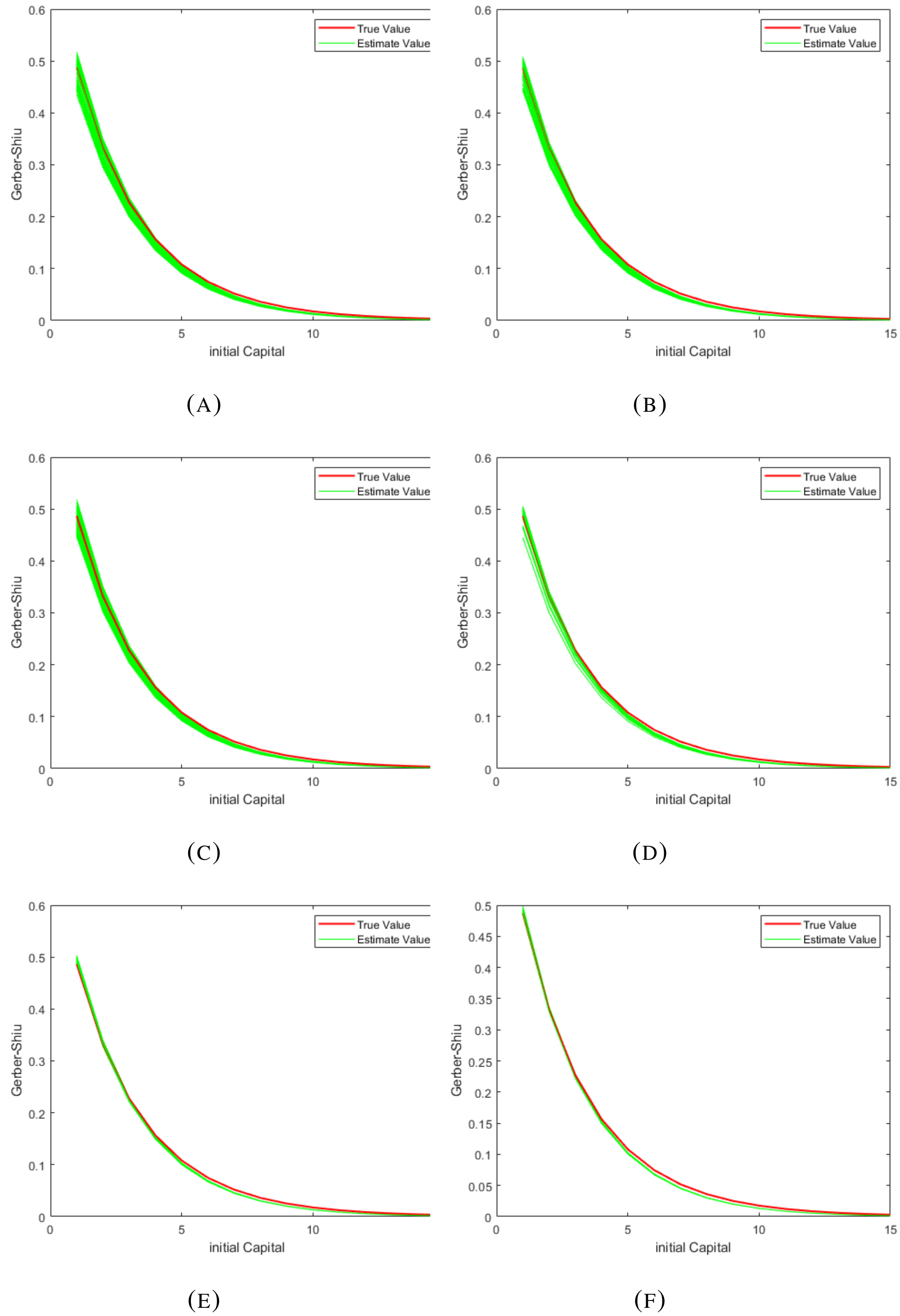

FIGURE 2. Beams for estimating the Gerber-Shiu function $\phi(u,b)$: 300 estimators in green, and the true value in bold red. (a) q=1000; (b) q=2000; (c) q=3000; (d) q=4000; (e) q=5000; (f) q=6000

7. CONCLUSION

In this paper, we have estimated the espected present value of dividend payments before ruin and the expected discounted penalty function under perturbed compound Poisson risk model with constant barrier dividend strategy. Suppose that we have the data set of claim sizes, claim numbers and the surplus flow levels, we construct our estimators based on the Fourier-Sinc series expansion method. Due to that we can use FFT algorithm to compute the coefficient in the Fouirer-Sinc series, the computation of our estimators is very fast. We have derived theoritical errors and presented some simulation results to show the effectiveness of our estimators.

A future resaerch could be to compare this method with the Fouirer Cosine methods which also uses Fourier transform. Moreover, one can think on how to estimatimate the expected present dividend payments and the Gerber-Shiu function under threshold dividend strategy.

ACKNOWLEDGMENTS

The Authors would like to thank the Pan African University Institute of Basic Sciences Technology and Innovation (PAUSTI) for their financial assistance.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] B. De Finetti, Su un'impostazione alternativa della teoria collettiva del rischio, in: Transactions of the XVth International Congress of Actuaries, Volume 2, pp. 433–443, New York, 1957.

[2] Z. Liu, P. Chen, Dividend payments until draw-down time for risk models driven by spectrally negative levy processes, Commun. Stat.-Simul. Comput. 51 (2022), 7226–7245.

[3] Y. Zhao, H. Dong, W. Zhong, Equilibrium dividend strategies for spectrally negative levy processes with time value of ruin and random time horizon, Commun. Stat.-Theory Methods, 51 (2022), 4757–4780.

[4] J. Pecaric, A. Perusic, A. Vukelic, Generalisations of Steffensen's inequality via Fink identity and related results, Adv. Inequal. Appl. 2014 (2014), 9.

[5] R.L. Loeffen, On optimality of the barrier strategy in de finetti's dividend problem for spectrally negative levy processes, Ann. Appl. Probab. 18 (2008), 1669–1680.

[6] H. You, J. Guo, J. Jiang, Interval estimation of the ruin probability in the classical compound Poisson risk model, Comput. Stat. Data Anal. 144 (2020), 106890.

[7] Y. Shimizu, Z. Zhang, Asymptotically normal estimators of the ruin probability for levy insurance surplus from discrete samples, Risks, 7 (2019), 37.

[8] F. Dussap, Nonparametric estimation of the expected discounted penalty function in the compound poisson model, Electron. J. Stat. 16 (2022), 2124–2174.

[9] W. Su, Y. Yong, Z. Zhang, Estimating the gerber–shiu function in the perturbed compound poisson model by laguerre series expansion, J. Math. Anal. Appl. 469 (2019), 705–729.

[10] W. Su, Y. Wang, Estimating the gerber-shiu function in levy insurance risk model by fourier-cosine series expansion, Mathematics, 9 (2021), 1402.

[11] Y. Huang, W. Yu, Y. Pan, C. Cui, Estimating the gerber-shiu expected discounted penalty function for levy risk model, Discr. Dyn. Nat. Soc. 2019 (2019), 3607201.

[12] J. Xie, Z. Zhang, Statistical estimation for some dividend problems under the compound poisson risk model, Insurance: Math. Econ. 95 (2020), 101–115.

[13] Y. Yang, J. Xie, Z. Zhang, Nonparametric estimation of some dividend problems in the perturbed compound poisson model, Probab. Eng. Inform. Sci. 37 (2023), 418–441.

[14] F. Lundberg, F.I. Riskutjämning, Teori. II: Statistik (Insurance technical smoothing of risks) F. Englunds Boktryckeri AB, Stockholm, (1926).

[15] C. Constantinescu, G. Samorodnitsky, W. Zhu, Ruin probabilities in classical risk models with gamma claims, Scandinavian Actuarial J. 2018 (2018), 555–575.

[16] F. Lundberg, Approximerad framställning af sannolikhetsfunktionen: Återförsäkring af kollektivrisker (Doctoral dissertation, Almqvist & Wiksell), (1903).

[17] Z. Zhang, Estimating the gerber–shiu function by fourier–sinc series expansion, Scandinavian Actuarial J. 2017 (2017), 898–919.

[18] M. Egami, K. Yamazaki, Phase-type fitting of scale functions for spectrally negative levy processes, J. Comput. Appl. Math. 264 (2014), 1–22.

[19] Z. Zhang, Z. Cui, Laguerre series expansion for scale functions and its applications in risk theory, Technical report, Working Paper, 2019.

[20] Y. Shimizu, Z. Zhang, Asymptotically normal estimators of the ruin probability for levy insurance surplus from discrete samples, Risks, 7 (2019), 37.

[21] A.W. Van der Vaart, Asymptotic statistics, Volume 3, Cambridge University Press, 2000.