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OTHER POSITIONS OF PARAMETERS IN THE ASYMPTOTIC EXPANSIONS OF KNOCK-IN BARRIER OPTION PRICES

DENNIS G. LLEMIT

Department of Mathematics, Adamson University, 900 San Marcelino St., Ermita, 1000 Manila, Philippines

Abstract. Joshi's general method for computing the asymptotic expansions of european options was conceived by placing the strike price at the center of the binomial tree [6]. This set-up showed that the errors in approximating the continuous pricing models by discrete ones through asymptotic expansion is of order $\frac{1}{n}$. In this paper, we find other positions, aside from the center, for the parameters K (strike price) and B (barrier level) in the asymptotic expansions of Knock-in barrier option prices under Joshi's general method. This has been shown to be possible for the case of an Up-and-In Put (UIP) barrier option in the paper done by Llemit and Escaner [1].

Keywords: knock-in barrier option, asyptotic expansion, Joshi's general method.

2000 AMS Subject Classification: 47H17; 47H05; 47H09

1. Introduction

Knock-in barrier options are derivative contracts which activate only whenever the underlying instrument has breached a certain treshhold called the barrier level. Together with their counterparts, the Knock-out barrier options, they are usually traded in over-the-counter (OTC) markets and are interesting since they are cheaper compared to vanilla

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options.

The convergence of discrete pricing models, especially the Cox-Ross-Rubinstein (CR-R) ,to the famous continuous model, Black-Scholes, has been the subject of numerous studies. Hsia [3] provided the most elegant and comprehensive proof for this convergence. Diener and Diener[2] computed the asymptotic convergence of a Down-and-In Call (DIC) barrier option using a technique called bounded coefficients and showed that the order is $\frac{1}{\sqrt{n}}$. Mark Joshi[6] developed a general method in computing the asymptotic expansions of general vanilla options and showed that the convergence is of order $\frac{1}{n}$. Llemit and Escaner[1] applied the general method of Joshi and showed that the convergence is consistent for an Up-and-In Put (UIP) barrier option. However, they found out that the parameters B and K are not limited to the center of the binomial tree. This result is different from the assumptions of the general method of Joshi.

In this paper, we examine other types of Knock-in barrier options and determine whether the results obtained in [1]can be replicated. These are the UIC (Up-and-In Call), DIC (Down-and-In Call), and the DIP (Down-and-In Put). We begin by listing the pricing formulas developed by Levitan, Mitchell and Taylor[8]:

For an Up-and-In Call (UIC) barrier option, we have

If $x > m$, then

$$UIC_0 = e^{-rT} \sum_{j=\lceil \frac{n+x}{2} \rceil}^{n} \binom{n}{j} p^j (1-p)^{n-j} \left(S_0 u^n d^{n-j} - K \right). \tag{1}$$

If $x \leq m$ and when $(n+m)$ is odd, then

$$\begin{aligned} UIC_0 &= e^{-rT} \left[\sum_{j=\lceil \frac{n+x}{2} \rceil}^{\lfloor \frac{n+m}{2} \rfloor} \binom{n}{j-m} p^j (1-p)^{n-j} \left(S_0 u^n d^{n-j} - K \right) \right. \\ &\quad \left. + \sum_{j=\lceil \frac{n+m}{2} \rceil}^{n} \binom{n}{j} p^j (1-p)^{n-j} \left(S_0 u^n d^{n-j} - K \right) \right]. \end{aligned} \tag{2}$$

If $x \leq m$ and when $(n+m)$ is even, then

$$\begin{aligned} UIC_0 &= e^{-rT}\left[\sum_{j=\left\lceil\frac{n+x}{2}\right\rceil}^{\frac{n+m}{2}-1}\binom{n}{j-m}p^j(1-p)^{n-j}\left(S_0u^nd^{n-j}-K\right)\right. \\ &\left.+\sum_{j=\frac{n+m}{2}}^{\left\lfloor\frac{n+x}{2}\right\rfloor}\binom{n}{j}p^j(1-p)^{n-j}\left(S_0u^nd^{n-j}-K\right)\right]. \qquad (3) \end{aligned}$$

For a Down-and-In Call (DIC) barrier option, we have

If $x \leq m$, then

$$DIC_0 = e^{-rT}\sum_{j=\left\lfloor\frac{n-x}{2}\right\rfloor}^{n-m}\binom{n}{m+j}p^j(1-p)^{n-j}\left(S_0u^nd^{n-j}-K\right). \qquad (4)$$

If $x > m$ and when $(n-m)$ is odd, then

$$\begin{aligned} DIC_0 &= e^{-rT}\left[\sum_{j=\left\lceil\frac{n-x}{2}\right\rceil}^{\left\lfloor\frac{n-m}{2}\right\rfloor}\binom{n}{j}p^j(1-p)^{n-j}\left(S_0u^nd^{n-j}-K\right)\right. \\ &\left.+\sum_{j=\left\lceil\frac{n-m}{2}\right\rceil}^{n-m}\binom{n}{m+j}p^j(1-p)^{n-j}\left(S_0u^nd^{n-j}-K\right)\right]. \qquad (5) \end{aligned}$$

If $x > m$ and when $(n-m)$ is even, then

$$\begin{aligned} DIC_0 &= e^{-rT}\left[\sum_{j=\left\lceil\frac{n+x}{2}\right\rceil}^{\frac{n-m}{2}-1}\binom{n}{j}p^j(1-p)^{n-j}\left(S_0u^nd^{n-j}-K\right)\right. \\ &\left.+\sum_{j=\frac{n-m}{2}}^{n-m}\binom{n}{m+j}p^j(1-p)^{n-j}\left(S_0u^nd^{n-j}-K\right)\right]. \qquad (6) \end{aligned}$$

For a Down-and-In Put (DIP) barrier option, we have

If $x > m$, then

$$DIP_0 = e^{-rT}\sum_{j=0}^{\left\lfloor\frac{n+x}{2}\right\rfloor}\binom{n}{j}p^j(1-p)^{n-j}\left(K-S_0u^nd^{n-j}\right). \qquad (7)$$

If $x \leq m$ and when $(n-m)$ is odd, then

$$
\begin{aligned}
DIP_0 &= e^{-rT}\left[\sum_{j=0}^{\lfloor\frac{n-m}{2}\rfloor}\binom{n}{j}p^j(1-p)^{n-j}\left(K-S_0u^nd^{n-j}\right)\right.\\
&\left.+\sum_{j=\lceil\frac{n-m}{2}\rceil}^{\lfloor\frac{n+x}{2}\rfloor}\binom{n}{m+j}p^j(1-p)^{n-j}\left(K-S_0u^nd^{n-j}\right)\right]. \qquad (8)
\end{aligned}
$$

If $x \leq m$ and when $(n-m)$ is even, then

$$
\begin{aligned}
DIP_0 &= e^{-rT}\left[\sum_{j=0}^{\frac{n-m}{2}-1}\binom{n}{j}p^j(1-p)^{n-j}\left(K-S_0u^nd^{n-j}\right)\right.\\
&\left.+\sum_{j=\frac{n-m}{2}}^{\lfloor\frac{n+x}{2}\rfloor}\binom{n}{m+j}p^j(1-p)^{n-j}\left(K-S_0u^nd^{n-j}\right)\right]. \qquad (9)
\end{aligned}
$$

where

- m - number of up or down movements necessary to breach the barrier level B
- x - number of up or down movements necessary to breach the strike level K
- p - risk-neutral probability
- r - risk-free interest rate
- u - up factor
- d - down factor
- n - time steps at maturity.

2.Transformations

Since these formulas are discrete, we cannot readily apply asymptotic expansion. Thus, we will use a lemma to make them analytic.

Lemma 1. *Let n and k be integers where $0 \leq k \leq n$ and p is the probability of an up movement.*

$$\sum_{j=k}^{n}\binom{n}{j}p^j(1-p)^{n-j} = k\binom{n}{k}\int_0^p y^{k-1}(1-y)^{n-k}\,dy. \qquad (10)$$

Using this formula we get their respective integral forms. For equation (1), we have

$$\begin{aligned} UIC_0 &= S_0\left(\left\lceil\frac{n+x}{2}\right\rceil\right)\binom{n}{\lceil\frac{n+x}{2}\rceil}\int_0^q y^{\lceil\frac{n+x}{2}\rceil-1}(1-y)^{n-\lceil\frac{n+x}{2}\rceil}\,dy \\ &\quad -Ke^{-rT}\left(\left\lceil\frac{n+x}{2}\right\rceil\right)\binom{n}{\lceil\frac{n+x}{2}\rceil}\int_0^p y^{\lceil\frac{n+x}{2}\rceil-1}(1-y)^{n-\lceil\frac{n+x}{2}\rceil}\,dy \end{aligned}$$

Similarly, for equation (2), we have

$$\begin{aligned} UIC_0 &= S_0\left(\frac{q}{1-q}\right)^{n-\lfloor\frac{n+m}{2}\rfloor}\left(n+\left\lceil\frac{n+x}{2}\right\rceil-\left\lfloor\frac{n+m}{2}\right\rfloor\right)\binom{n}{n+\lceil\frac{n+x}{2}\rceil-\lfloor\frac{n+m}{2}\rfloor} \\ &\quad\times\int_0^q y^{n+\lceil\frac{n+x}{2}\rceil-\lfloor\frac{n+m}{2}\rfloor-1}(1-y)^{\lfloor\frac{n+m}{2}\rfloor-\lceil\frac{n+x}{2}\rceil}\,dy \\ &\quad -Ke^{-rT}\left(\frac{p}{1-p}\right)^{n-\lfloor\frac{n+m}{2}\rfloor}\left(n+\left\lceil\frac{n+x}{2}\right\rceil-\left\lfloor\frac{n+m}{2}\right\rfloor\right)\binom{n}{n+\lceil\frac{n+x}{2}\rceil-\lfloor\frac{n+m}{2}\rfloor} \\ &\quad\times\int_0^p y^{n+\lceil\frac{n+x}{2}\rceil-\lfloor\frac{n+m}{2}\rfloor-1}(1-y)^{\lfloor\frac{n+m}{2}\rfloor-\lceil\frac{n+x}{2}\rceil}\,dy \\ &\quad +S_0\left(\left\lceil\frac{n+m}{2}\right\rceil\right)\binom{n}{\lceil\frac{n+m}{2}\rceil}\int_0^q y^{\lceil\frac{n+m}{2}\rceil-1}(1-y)^{n-\lceil\frac{n+m}{2}\rceil}\,dy \\ &\quad -Ke^{-rT}\left(\left\lceil\frac{n+m}{2}\right\rceil\right)\binom{n}{\lceil\frac{n+m}{2}\rceil}\int_0^p y^{\lceil\frac{n+m}{2}\rceil-1}(1-y)^{n-\lceil\frac{n+m}{2}\rceil}\,dy \end{aligned}$$

and for equation (3),

$$\begin{aligned} UIC_0 &= S_0\left(\frac{q}{1-q}\right)^{\frac{n+m}{2}-1-n}\left(n+\left\lceil\frac{n+x}{2}\right\rceil+1-\frac{n+m}{2}\right)\binom{n}{n+\lceil\frac{n+x}{2}\rceil+1-\frac{n+m}{2}} \\ &\quad\times\int_0^q y^{n+\lceil\frac{n+x}{2}\rceil-\frac{n+m}{2}}(1-y)^{\frac{n+m}{2}-1-\lceil\frac{n+x}{2}\rceil}\,dy \\ &\quad -Ke^{-rT}\left(\frac{p}{1-p}\right)^{\frac{n+m}{2}-1-n}\left(n+\left\lceil\frac{n+x}{2}\right\rceil+1-\frac{n+m}{2}\right)\binom{n}{n+\lceil\frac{n+x}{2}\rceil+1-\frac{n+m}{2}} \\ &\quad\times\int_0^p y^{n+\lceil\frac{n+x}{2}\rceil-\frac{n+m}{2}}(1-y)^{\frac{n+m}{2}-1-\lceil\frac{n+x}{2}\rceil}\,dy. \end{aligned}$$

For equations (4),(5),and (6), we have

$$\begin{aligned} DIC_0 &= S_0\left(\frac{q}{1-q}\right)^m \left(m+\left\lceil\frac{n-x}{2}\right\rceil\right)\binom{n}{m+\lceil\frac{n-x}{2}\rceil} \\ &\times\int_0^q y^{m+\lceil\frac{n-x}{2}\rceil-1}(1-y)^{n-m-\lceil\frac{n-x}{2}\rceil}\,dy \\ &-Ke^{-rT}\left(\frac{p}{1-p}\right)^m \left(m+\left\lceil\frac{n-x}{2}\right\rceil\right)\binom{n}{m+\lceil\frac{n-x}{2}\rceil} \\ &\times\int_0^p y^{m+\lceil\frac{n-x}{2}\rceil-1}(1-y)^{n-m-\lceil\frac{n-x}{2}\rceil}\,dy, \end{aligned}$$

$$\begin{aligned} DIC_0 &= S_0\left(\left\lceil\frac{n-x}{2}\right\rceil\right)\binom{n}{\lceil\frac{n-x}{2}\rceil}\int_0^q y^{\lceil\frac{n-x}{2}\rceil-1}(1-y)^{n-\lceil\frac{n-x}{2}\rceil}\,dy \\ &-S_0\left(\left\lceil\frac{n-m}{2}\right\rceil\right)\binom{n}{\lceil\frac{n-m}{2}\rceil}\int_0^q y^{\lceil\frac{n-m}{2}\rceil-1}(1-y)^{n-\lceil\frac{n-m}{2}\rceil}\,dy \\ &-Ke^{-rT}\left(\left\lceil\frac{n-x}{2}\right\rceil\right)\binom{n}{\lceil\frac{n-x}{2}\rceil}\int_0^p y^{\lceil\frac{n-x}{2}\rceil-1}(1-y)^{n-\lceil\frac{n-x}{2}\rceil}\,dy \\ &+Ke^{-rT}\left(\left\lceil\frac{n-m}{2}\right\rceil\right)\binom{n}{\lceil\frac{n-m}{2}\rceil}\int_0^q y^{\lceil\frac{n-m}{2}\rceil-1}(1-y)^{n-\lceil\frac{n-m}{2}\rceil}\,dy \\ &+S_0\left(\frac{q}{1-q}\right)^m \left(m+\left\lceil\frac{n-m}{2}\right\rceil\right)\binom{n}{m+\lceil\frac{n-m}{2}\rceil} \\ &\times\int_0^q y^{m+\lceil\frac{n-m}{2}\rceil-1}(1-y)^{n-m-\lceil\frac{n-m}{2}\rceil}\,dy \\ &-Ke^{-rT}\left(\frac{p}{1-p}\right)^m \left(m+\left\lceil\frac{n-m}{2}\right\rceil\right)\binom{n}{m+\lceil\frac{n-m}{2}\rceil} \\ &\times\int_0^p y^{m+\lceil\frac{n-m}{2}\rceil-1}(1-y)^{n-m-\lceil\frac{n-m}{2}\rceil}\,dy, \end{aligned}$$

and

$$\begin{aligned} DIC_0 &= S_0 \left(\left\lceil \frac{n-x}{2} \right\rceil \right) \binom{n}{\lceil \frac{n-x}{2} \rceil} \int_0^q y^{\lceil \frac{n-x}{2} \rceil - 1} (1-y)^{n - \lceil \frac{n-x}{2} \rceil} \, dy \\ &- S_0 \left(\frac{n-m}{2} \right) \binom{n}{\frac{n-m}{2}} \int_0^q y^{\frac{n-m}{2} - 1} (1-y)^{\frac{n+m}{2}} \, dy \\ &- K e^{-rT} \left(\left\lceil \frac{n-x}{2} \right\rceil \right) \binom{n}{\lceil \frac{n-x}{2} \rceil} \int_0^p y^{\lceil \frac{n-x}{2} \rceil - 1} (1-y)^{n - \lceil \frac{n-x}{2} \rceil} \, dy \\ &+ K e^{-rT} \left(\frac{n-m}{2} \right) \binom{n}{\frac{n-m}{2}} \int_0^p y^{\frac{n-m}{2} - 1} (1-y)^{\frac{n+m}{2}} \, dy \\ &+ S_0 \left(\frac{q}{1-q} \right)^m \left(\frac{m+n}{2} \right) \binom{n}{\frac{m+n}{2}} \int_0^q y^{\frac{m+n}{2} - 1} (1-y)^{n - \frac{m+n}{2}} \, dy \\ &- K e^{-rT} \left(\frac{p}{1-p} \right)^m \left(\frac{m+n}{2} \right) \binom{n}{\frac{m+n}{2}} \int_0^p y^{\frac{m+n}{2} - 1} (1-y)^{n - \frac{m+n}{2}} \, dy. \end{aligned}$$

And for equations (7), (8), and (9),we have

$$\begin{aligned} DIP_0 &= S_0 \left(\left\lceil \frac{n+x}{2} \right\rceil \right) \binom{n}{\lceil \frac{n+x}{2} \rceil} \int_0^q y^{\lceil \frac{n+x}{2} \rceil - 1} (1-y)^{n - \lceil \frac{n+x}{2} \rceil} dy \\ &- K e^{-rT} \left(\left\lceil \frac{n+x}{2} \right\rceil \right) \binom{n}{\lceil \frac{n+x}{2} \rceil} \int_0^p y^{\lceil \frac{n+x}{2} \rceil - 1} (1-y)^{n - \lceil \frac{n+x}{2} \rceil} dy, \end{aligned}$$

$$\begin{aligned} DIP_0 &= S_0 \left(\left\lceil \frac{n-m}{2} \right\rceil \right) \binom{n}{\lceil \frac{n-m}{2} \rceil} \int_0^q y^{\lceil \frac{n-m}{2} \rceil - 1} (1-y)^{n - \lceil \frac{n-m}{2} \rceil} \, dy \\ &- K e^{-rT} \left(\left\lceil \frac{n-m}{2} \right\rceil \right) \binom{n}{\lceil \frac{n-m}{2} \rceil} \int_0^p y^{\lceil \frac{n-m}{2} \rceil - 1} (1-y)^{n - \lceil \frac{n-m}{2} \rceil} \, dy \\ &+ K e^{-rT} \left(\frac{p}{1-p} \right)^{n - \lfloor \frac{n+x}{2} \rfloor} \left(n - \left\lfloor \frac{n+x}{2} \right\rfloor + \left\lceil \frac{n-m}{2} \right\rceil \right) \binom{n}{n - \lfloor \frac{n+x}{2} \rfloor + \lceil \frac{n-m}{2} \rceil} \\ &\times \int_0^p y^{n - \lfloor \frac{n+x}{2} \rfloor + \lceil \frac{n-m}{2} \rceil - 1} (1-y)^{\lfloor \frac{n+x}{2} \rfloor - \lceil \frac{n-m}{2} \rceil} \, dy \\ &- S_0 \left(\frac{q}{1-q} \right)^{n - \lfloor \frac{n+x}{2} \rfloor} \left(n - \left\lfloor \frac{n+x}{2} \right\rfloor + \left\lceil \frac{n-m}{2} \right\rceil \right) \binom{n}{n - \lfloor \frac{n+x}{2} \rfloor + \lceil \frac{n-m}{2} \rceil} \\ &\times \int_0^q y^{n - \lfloor \frac{n+x}{2} \rfloor + \lceil \frac{n-m}{2} \rceil - 1} (1-y)^{\lfloor \frac{n+x}{2} \rfloor - \lceil \frac{n-m}{2} \rceil} \, dy, \end{aligned}$$

and

$$\begin{aligned}
DIP_0 &= S_0 \left(\frac{n-m}{2}\right) \binom{n}{\frac{n-m}{2}} \int_0^q y^{\frac{n-m}{2}-1}(1-y)^{\frac{n+m}{2}}\,dy \\
&\quad -Ke^{-rT} \left(\frac{n-m}{2}\right) \binom{n}{\frac{n-m}{2}} \int_0^p y^{\frac{n-m}{2}-1}(1-y)^{\frac{n+m}{2}}\,dy \\
&\quad +Ke^{-rT} \left(\frac{p}{1-p}\right)^{n-\left\lfloor\frac{n+x}{2}\right\rfloor} \left(n-\left\lfloor\frac{n+x}{2}\right\rfloor+\frac{n-m}{2}\right) \binom{n}{n-\left\lfloor\frac{n+x}{2}\right\rfloor+\frac{n-m}{2}} \\
&\quad \times \int_0^p y^{n-\left\lfloor\frac{n+x}{2}\right\rfloor+\frac{n-m}{2}-1}(1-y)^{\left\lfloor\frac{n+x}{2}\right\rfloor-\frac{n-m}{2}}\,dy \\
&\quad -S_0 \left(\frac{q}{1-q}\right)^{n-\left\lfloor\frac{n+x}{2}\right\rfloor} \left(n-\left\lfloor\frac{n+x}{2}\right\rfloor+\frac{n-m}{2}\right) \binom{n}{n-\left\lfloor\frac{n+x}{2}\right\rfloor+\frac{n-m}{2}} \\
&\quad \times \int_0^q y^{n-\left\lfloor\frac{n+x}{2}\right\rfloor+\frac{n-m}{2}-1}(1-y)^{\left\lfloor\frac{n+x}{2}\right\rfloor-\frac{n-m}{2}}\,dy
\end{aligned}$$

3.Results

We utilize Joshi's general method by equating all the exponents in the integrals and setting $n = 2N+1$. Hence, we generate their general forms. For the UIC, we have

$$UIC_0 = \alpha\,(N+1)\binom{2N+1}{N+1}\left(S_0\int_0^q y^N(1-y)^N\,dy - Ke^{-rT}\int_0^p y^N(1-y)^N\,dy\right)$$

where α is either 1 or 2.

The general form of the DIC is

$$\begin{aligned}
DIC_0 &= (N+1)\binom{2N+1}{N+1}\left(S_0\left(\frac{q}{1-q}\right)^m \int_0^q y^N(1-y)^N\,dy\right. \\
&\quad \left. -Ke^{-rT}\left(\frac{p}{1-p}\right)^m \int_0^p y^N(1-y)^N\,dy\right),
\end{aligned}$$

while that of the DIP is

$$\begin{aligned} DIP_0 &= (N+1)\binom{2N+1}{N+1}\Bigg[\Bigg(S_0\int_0^q yN(1-y)^N\,dy \\ &\quad -Ke^{-rT}\int_0^p y^N(1-y)^N\,dy\Bigg)+\Bigg(Ke^{-rT}\left(\frac{p}{1-p}\right)^0 \\ &\quad \times\int_0^p y^N(1-y)^N\,dy - S_0\left(\frac{q}{1-q}\right)^0\int_0^q y^N(1-y)^N\,dy\Bigg)\Bigg] \\ &= 0 \end{aligned}$$

A significant result comes out and is stated as a theorem.

Theorem 1. *Let* x *be the number of down movements needed to breach the strike price* K, *m the number of down movements needed to breach the barrier level* B *and* n *the number of time steps. Then the order of convergence of a DIC barrier option under Joshi's general method holds if either of the following is true:*

(1) *If (n-x) is odd, x=m=0.*
(2) *If (n-x) is even, x=m=1.*

Proof: After applying Joshi's general method, we obtain the following equations:

If (n-x) is odd, then

$$m = \frac{x}{2}$$

If (n-x) is even, then

$$m = \frac{x+1}{2}$$

and susbsequently,

$$n = 2\left\lceil \frac{n-x}{2} \right\rceil - 1$$
$$n = 2\left\lceil \frac{n-m}{2} \right\rceil - 1$$
$$n = 2m + 2\left\lceil \frac{n-m}{2} \right\rceil - 1$$
$$n = 2\left(\frac{n+m}{2} \right) - 1$$

Solving them gives us two consistent solution set $x = 0, 1$ and $m = 0, 1$. We need to check which of these solutions agree with the pricing formula. We set-up a table for easier inspection.

(n-x)	x	Implied Nature of n	Assumed Nature for (n-m)	Implied Nature of m	Admissible Value for m	Calculated Value of m	Remark
Odd	0	Odd	Odd	Even	0	0	Consistent
Odd	0	Odd	Even	Odd	1	0	Inconsistent
Odd	1	Even	Odd	Odd	1	1/2	Inconsistent
Odd	1	Even	Even	Even	0	1/2	Inconsistent
Even	0	Even	Odd	Odd	1	1/2	Inconsistent
Even	0	Even	Even	Even	0	1/2	Inconsistent
Even	1	Odd	Odd	Even	0	1	Inconsistent
Even	1	Odd	Even	Odd	1	1	Consistent

As we can see from the table, only the cases $x = m = 0$ and $x = m = 1$ are consistent. Therefore, the theorem has been proven.

4.Conclusions

From the calculations, Joshi's general method does not work for a DIP barrier option since the pricing formula was annihilated upon its application. On the other hand, the general method works for both the UIC and DIC barrier options, but the general form of the UIC does not give consistent locations for the positions of the parameters B and K.

Thus, only the DIC option produces results which are different from the assumptions of the general method and similar to the results obtained in [1]. That is, the barrier level B and the strike price K can be located one node below the center of the binomial tree.

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